Motion in Plane

MOTION IN A PLANE

Vector → Component of Vector
Vector → Magnitude of Vector

$\vec{F} = F_x\hat{i} + F_y\hat{j}$

$|\vec{F}| = \sqrt{F_x^2 + F_y^2}$

2-D Motion - (1-D) x-axis
+
(1-D) y-axis

Position Vector in Plane

$\vec{r} = x\hat{i} + y\hat{j}$

diffn w.r.t time

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

diffn w.r.t time

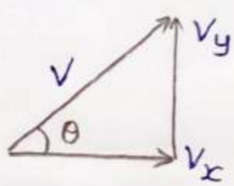
$$\frac{d\vec{V}}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j}$$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

$\tan\theta = \dfrac{V_y}{V_x}$

direction of motion from x-axis



If acceleration is constant we will use eqn of motion

$2D \rightarrow [1D] + [1D]$

$$\vec{V_x} = \vec{u_x} + \vec{a_x}t$$
$$\vec{V_y} = \vec{u_y} + \vec{a_y}t$$
$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$
$$|\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

$$x = u_x t + \frac{1}{2}a_x t^2$$
$$y = u_y t + \frac{1}{2}a_y t^2$$

$$V_x^2 - u_x^2 = 2a_x x$$
$$V_y^2 - u_y^2 = 2a_y y$$

Que- If initial velo. of object $\vec{u} = 3\hat{i} + 4\hat{j}$ and after some time its $v = 4\hat{i} + 3\hat{j}$ then find

(i) change in magnitude of velocity

(ii) magnitude of change in velocity

(i) $|\vec{V_f}| - |\vec{V_i}|$

$5 - 5 = 0$

(ii) $\vec{\Delta V} = \vec{V_f} - \vec{V_i}$

$= 4\hat{i} + 3\hat{j} - (3\hat{i} + 4\hat{j})$

$\vec{\Delta V} = \hat{i} - \hat{j}$

$|\vec{\Delta V}| = \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{2}$

Que- x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$; where x and y. Acceleration of the particle at $t = 2s$

$x = 5t - 2t^2$

$\frac{dx}{dt} = V_x = 5 - 2(2t)$

$V_x = 5 - 4t$

$a_x = \frac{dV_x}{dt} = -4$

$y = 10t$

$\frac{dy}{dt} = V_y = 10$

$\frac{dV_y}{dt} = a_y = 0$

$\vec{a} = a_x\hat{i} + a_y\hat{j} = -4\hat{i} + 0$

$= -4\, m/s^2\, \hat{i}$

Que- A Particle has initial velocity $2\hat{i} + 3\hat{j}$ and acceleration $(0.3\hat{i} + 0.2\hat{j})$. Magnitude of velocity after 10 sec

$\vec{u} = 2\hat{i} + 3\hat{j}$

$u_x = 2 \qquad u_y = 3$

$a_x = 0.3 \qquad a_y = 0.2$

$v_x = u_x + a_x t \qquad v_y = 3 + 0.2 \times 10$

$= 2 + 0.3 \times 10 \qquad v_y = 3 + 2$

$= 2 + 3 \qquad v_y = 5\hat{j}$

$v_x = 5\hat{i}$

$v = 5\hat{i} + 5\hat{j}$

$|v| = 5\sqrt{2}$

Que- A particle moving in plane such that $y = \frac{x^2}{4}$ and $x = \frac{t}{2}$ then find speed at $t = 2$ sec

$y = \frac{x^2}{4}$

$x = \frac{t}{2}$

$y = \frac{t^2}{4 \times 4} = \frac{t^2}{16}$

$\frac{dx}{dt} = v_x = \frac{1}{2}\hat{i}$

$\frac{dy}{dt} = v_y = \frac{1}{16} 2t$

$v_y = \frac{t}{8}$

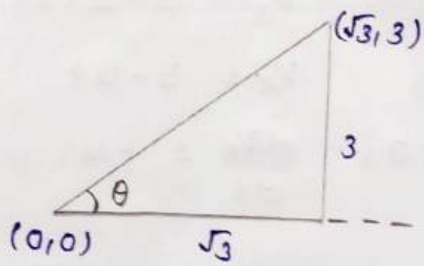
$\vec{v_y} = \left(\frac{t}{8}\right) = \frac{2}{8} = \frac{1}{4}\hat{j}$

$\vec{v} = \frac{1}{2}\hat{i} + \frac{1}{4}\hat{j}$

$|\vec{v}| = \sqrt{\frac{1}{4} + \frac{1}{16}}$

$= \sqrt{\frac{5}{16}} \Rightarrow \frac{\sqrt{5}}{4}$

Que- A particle starting from origin moves in straight line & reach at a point $(\sqrt{3}, 3)$ then Path makes an angle from x-axis



$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\theta = 60°$

Que- Object starts from the point $(2\hat{i}, 4\hat{j})$ m at $t=0$ with velocity $(5\hat{i}+4\hat{j})$ with constant acceleration $(4\hat{i}+4\hat{j})$ m/s^2. What is distance from particle from origin at $t = 2$ sec.

$t=0$

$x_i = (2,4)$ m

$\vec{v} = (5\hat{i}+4\hat{j})$ m/s

$\vec{a} = (4\hat{i}+4\hat{j})$ m/s^2

$x_f \ (x, y) = ?$

$$S_x = x_f - x_i = 5\times 2 + \frac{1}{2}\times 4(2)^2$$

$$x_f - 2 = 10 + 8 = 18$$

$$x_f = 20$$

$$y_f - y_i = 4\times 2 + \frac{1}{2}\times 4(2)^2$$

$$y_f - 4 = 16$$

$$y_f = 20$$

$(20, 20)$

$$\vec{r} = 20\hat{i} + 20\hat{j}$$

$$|\vec{r}| = \sqrt{(20)^2+(20)^2}$$

$$= 20\sqrt{2}$$

Que- Position of object $\vec{r} = (t^2 - 38t)\hat{i} + 2t^3\hat{j}$. find velocity and acceleration

$$\vec{r} = (t^2 - 38t)\hat{i} + 2t^3\hat{j}$$

$$\vec{v} = (2t - 38)\hat{i} + (6t^2)\hat{j}$$

$$\vec{a} = (2\hat{i} + 12t\,\hat{j})$$

Que- A particle starts from origin with velocity $3\hat{i}$ m/s and acceleration $(6\hat{i}+4\hat{j})$ then find x coordinates of particle when y coordinates is 32

$r = (0,0)$

$u = (3\hat{i}, 0)$ m/s

$a = (6\hat{i} + 4\hat{j})$ m/s^2

$x = 3t + \frac{1}{2}6t^2$

$x = 3\times4 + 3(4)^2$

$= 12 + 48$

$x = 60\,m$

$y = 0\times t + \frac{1}{2}4t^2$

$32 = 2t^2$

$t^2 = 16$

$t = \sqrt{16} = 4\,sec$

Que- object is moving with velocity $v = 3\sin(\omega t)\hat{i} + 3\cos(\omega t)\hat{j}$ then find distance moved by object in 2-sec

$\vec{V} = 3\sin(\omega t)\hat{i} + 3\cos(\omega t)\hat{j}$

$|\vec{V}| = \sqrt{[(3\sin(\omega t))]^2 + [3\cos(\omega t)]^2}$

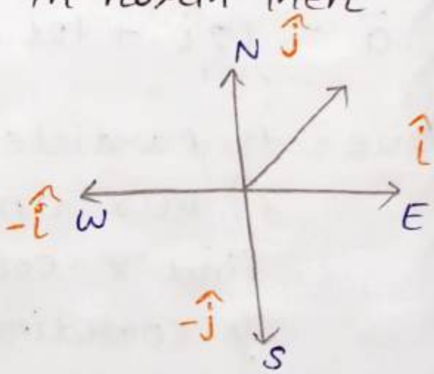
$\text{Speed} = \sqrt{3^2(\sin^2\omega t + \cos^2\omega t}$

$\text{speed} = 3\,m/s$

$\text{Distance} = \text{speed} \times \text{time}$

$3\times2 = 6m$

Que- object is moving in west with 5 m/s after 2 sec its velocity 5 m/s in north then find acceleration



$\vec{u_i} = -5\hat{i}$ (west)

$\vec{V_f} = +5\hat{j}$ (North)

$\vec{a} = \frac{\vec{V_f} - \vec{V_i}}{t} = \frac{5\hat{j} - (-5\hat{i})}{2}$

$= \frac{5\hat{i} + 5\hat{j}}{2} = \frac{5}{2}(\hat{i} + \hat{j})$

North East

$= \frac{5}{2}\sqrt{2} = \frac{5}{\sqrt{2}}\,m/s^2$

Que- Acceleration of Object $a = 2\hat{i} + 3t^2\hat{j}$, then find velocity at $t = 1$ sec it initial velocity of object is zero.

$$\frac{dv}{dt} = 2\hat{i} + 3t^2\hat{j}$$

$$\int_0^v dv = \int_0^1 2\,dt\,\hat{i} + \int_0^1 3t^2\,dt\,\hat{j}$$

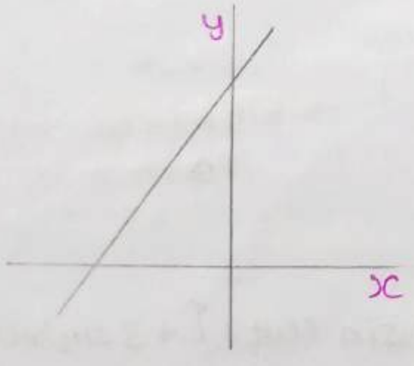
$$v = 2(t)_0^1\hat{i} + 3\left(\frac{t^3}{3}\right)_0^1\hat{j} = 2\hat{i} + 1\hat{j}$$

$$|v| = \sqrt{5}\ m/s$$

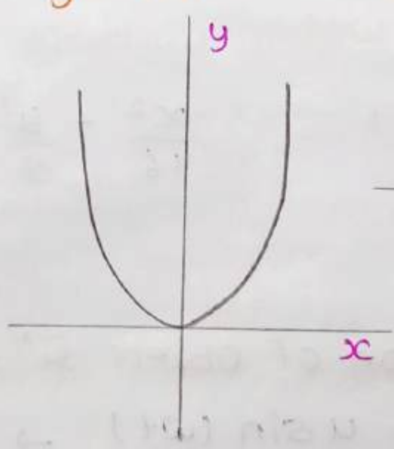
Equation of Trajectory

- Path followed by object can be determined by relation b/w x and y
- Relation b/w 'x' and 'y' position, represent path of object.

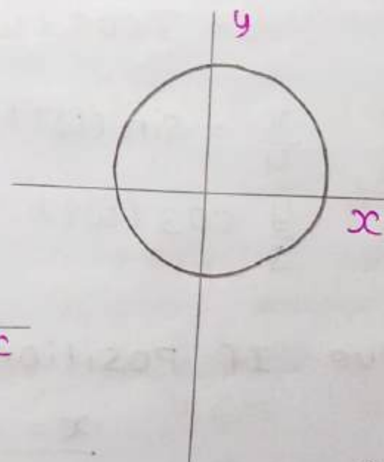
$y = 2x + 4$



$y = 2x^2$



$x^2 + y^2 = R^2$



Que- position of object at time 't' $\vec{r} = 2t\hat{i} + 4t^2\hat{j}$ then find equation of trajectory

$\vec{r} = 2t\hat{i} + 4t^2\hat{j}$

$x = 2t$

$t = \frac{x}{2}$

$y = 4t^2$

$y = \frac{4x^2}{4}$

$y = x^2 \rightarrow$ Parabolic Path

Que- If Position of object $\vec{r} = 3\sin(\omega t)\hat{i} + 3\cos(\omega t)\hat{j}$ then object is moving on

$x = 3\sin(\omega t)$ — (I)

$y = 3\cos(\omega t)$ — (II)

Add eqn (I) and (II)

$x^2 + y^2 = 9\sin^2\omega t + 9\cos^2\omega t$

$x^2 + y^2 = 9(1) \rightarrow$ circular Path

Que- If Position of object $\vec{r} = 4\sin(\omega t)\hat{i} + 3\cos(\omega t)\hat{j}$

$x = 4\sin(\omega t)$

$y = 3\cos(\omega t)$

$\frac{x}{4} = \sin(\omega t)$

$\frac{y}{3}\ \cos(\omega t)$

$\frac{x^2}{16} + \frac{y^2}{9} = 1 \rightarrow$ Elliptical Path

Que- If position of object $\vec{r} = 4\sin(\omega t)\hat{i} + 3\sin(\omega t)\hat{j}$

$$\frac{x = 4\sin(\omega t)}{y = 3\sin(\omega t)} \rightarrow \frac{x}{y} = \frac{4}{3}$$

$y = \frac{3x}{4} \rightarrow$ Straight line

Que- A particle moving with velocity $\vec{V} = y\hat{i} + x\hat{j}$ then find equation of trajectory

$V_x = y \qquad V_y = x$

$\frac{dx}{dt} = y$

$\frac{dy}{dt} = x$

$\rightarrow \int x\,dx = \int y\,dy$

$\rightarrow \frac{x^2}{2} = \frac{y^2}{2} + c$

$\boxed{x^2 = y^2 + c}$ → Eqn of Trajectory.

Que- Initial velocity of object $(4\hat{i} + 8\hat{j})$ m/s and acc. $a = -4$ m/s $\hat{j}$ then find velocity after $t = 3s$ and dispm in 3sec

$\vec{u_x} = 4$

$a_x = 0$

$\vec{V_x} = 4\hat{i}$

(constant)

$\vec{u_y} = 8$

$a_y = -4 m/s^2$

$\vec{V_y} = u_y + a_y t$

$= 8 - 4 \times 3$

$\vec{V_y} = -4\hat{j}$

$\vec{V} = \vec{V_x} + \vec{V_y}$

$= 4\hat{i} - 4\hat{j}$

Speed $|\vec{V}| = 4\sqrt{2}$ m/s

For dispm

$x = u_x t$ → uniform motion

$= 4 \times 3$

$x = 12m$

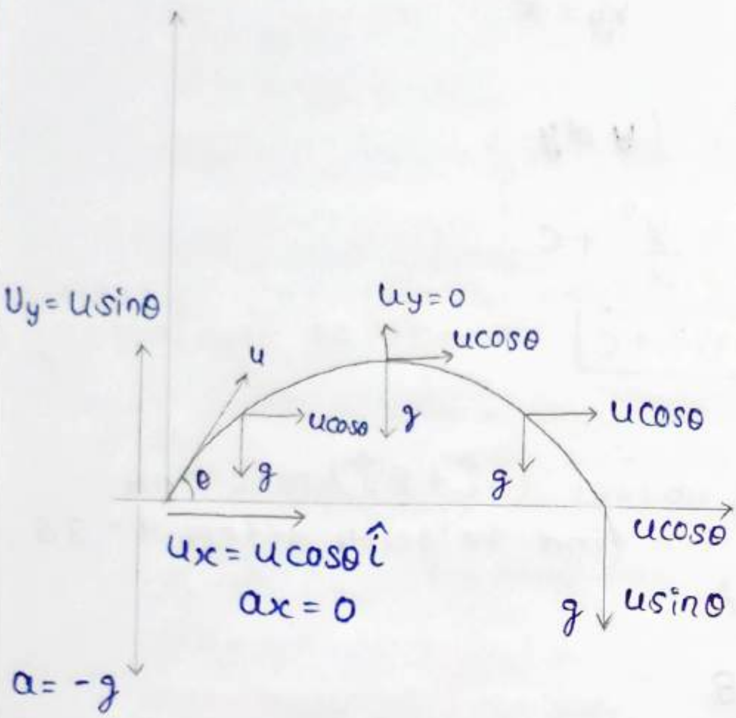
$y = u_y t + \frac{1}{2} a_y t^2$

$= 8 \times 3 - \frac{1}{2} 4 (3)^2$

$= 24 - 18$

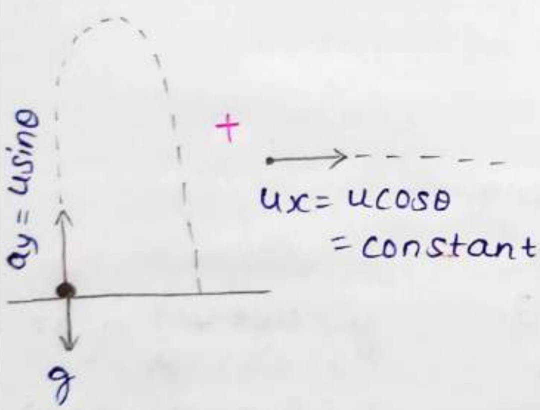
$= 6m$

Projectile Motion → 2D Non-uniform motion with uniform Acceleration



at $t=0$, $u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
• velocity along x-axis remain same. (uniform motion in 'x') $V_x = u_x = u\cos\theta$	• velocity will change along 'y' (Non-uniform motion in 'y') $V_y = u_y + a_y t$, $V_y = u\sin\theta - gt$
at 't' = t: $x = u_x t$, $x = u\cos\theta\, t$	at 't' = t: $y = u_y t + \frac{1}{2} a_y t^2$, $y = u\sin\theta t - \frac{1}{2} g t^2$

Projectile motion



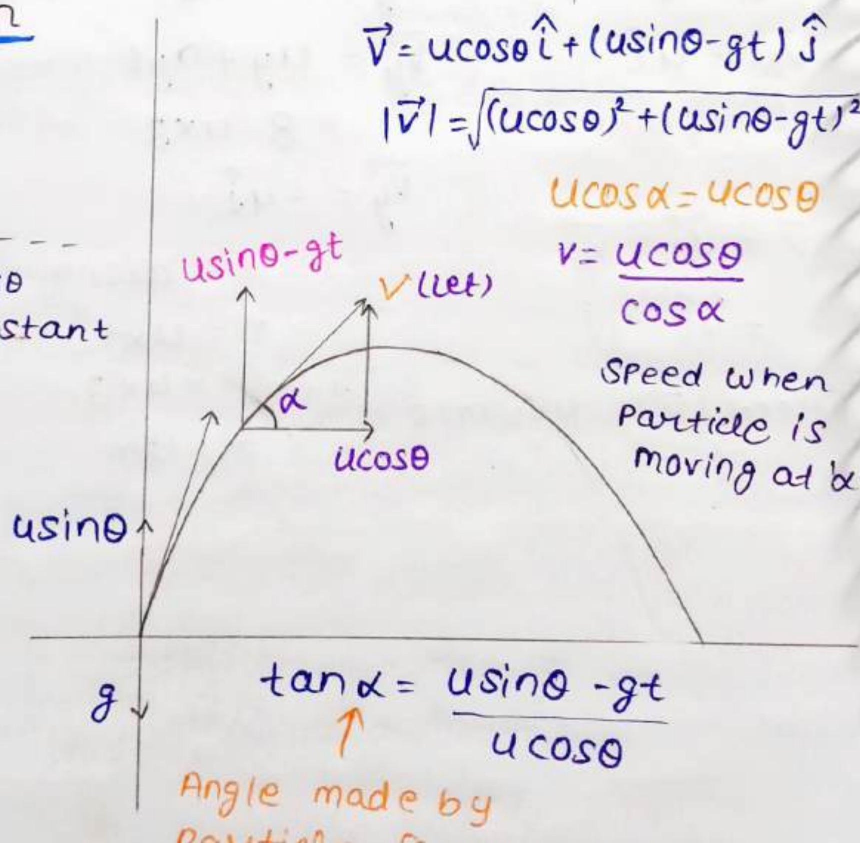
$$\vec{V} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

$$|\vec{V}| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

$$u\cos\alpha = u\cos\theta$$

$$v = \frac{u\cos\theta}{\cos\alpha}$$

Speed when particle is moving at 'α'



$$\tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$$

↑ Angle made by particle from x-axis after 't' time

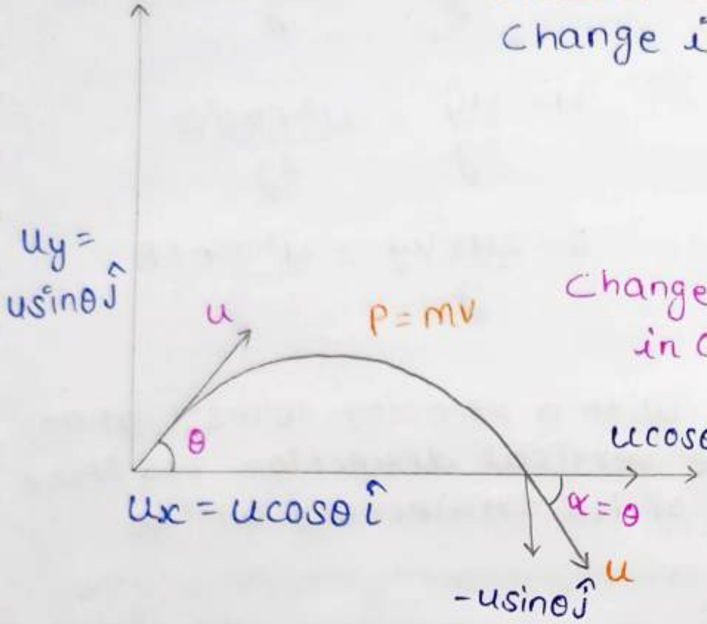
Velocity of Collision ≠ Velocity of Projection

Speed of Collision = Speed of Projection

Change in velocity $= \vec{V_f} - \vec{U_i}$

$= -u\sin\theta\hat{j} + u\cos\theta\hat{i} - u\cos\theta\hat{i} - u\sin\theta\hat{j}$

$\vec{\Delta V} = -2u\sin\theta\hat{j}$



Change in momentum in complete motion $= \Delta\vec{P} = m\Delta V$

$|\Delta\vec{P}| = m2u\sin\theta$

Avg. velocity in projectile (complete Journey)

$$\vec{u}_{Avg.} = \frac{\vec{u_i} + \vec{u_f}}{2} = u\cos\theta$$

$\vec{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

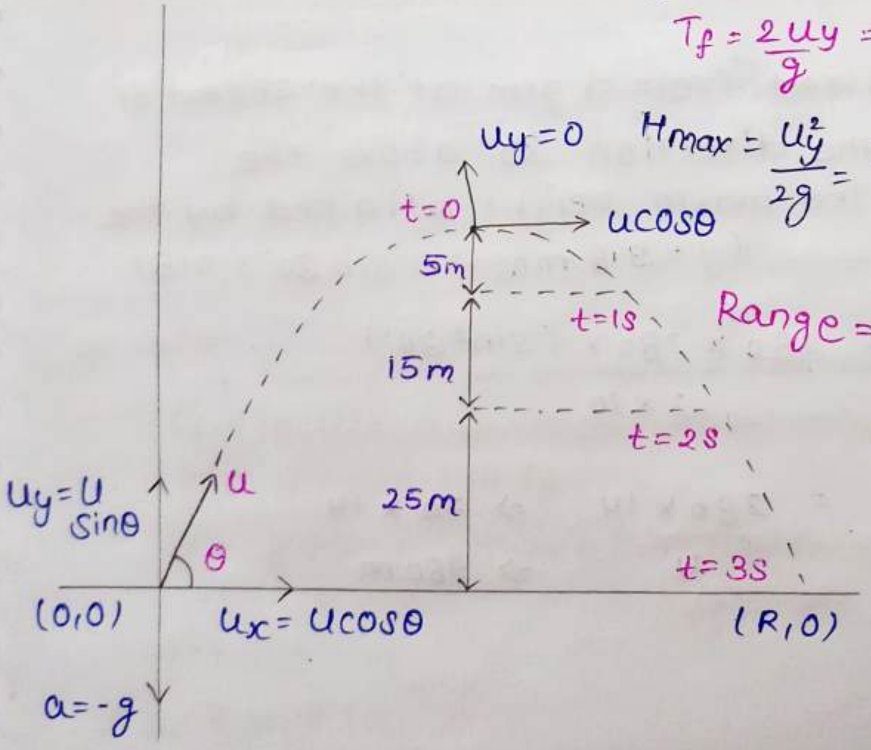
$\vec{v_f} = u\cos\theta\hat{i} - u\sin\theta\hat{j}$

Projectile motion - motion under gravity in 'y' + $(1\text{-}D)_x$ uniform motion

$$T_f = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

Time of flight does not depend on Horizontal velo. only depend on vertical velo.

$$H_{max} = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$



$$\text{Range} = u_x t_f = u_x\frac{2u_y}{g}$$

$$= \frac{2u_x u_y}{g} = \frac{2u\cos\theta \cdot u\sin\theta}{g}$$

$$R = \frac{u^2\sin(2\theta)}{g}$$

$\theta \rightarrow$ From Horizontal

$$T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

$$H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$

$$R = \frac{2u_x u_y}{g} = \frac{u^2\sin(2\theta)}{g}$$

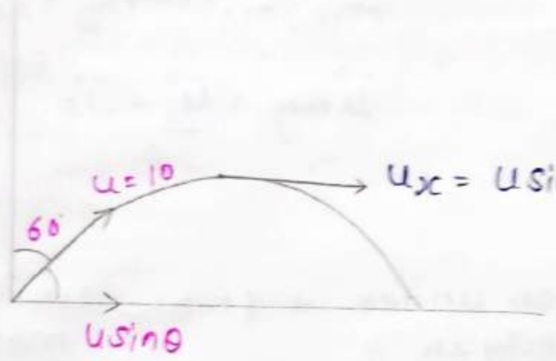
$\theta \rightarrow$ From vertical

$$T = \frac{2u_y}{g} = \frac{2u\cos\theta}{g}$$

$$H = \frac{u_y^2}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$R = \frac{2u_x u_y}{g} = \frac{u^2\sin 2\theta}{g}$$

Que- A ball is projected with a velocity, $10\ ms^{-1}$, at an angle of 60° with the vertical direction. Its speed at the highest point of its trajectory will be



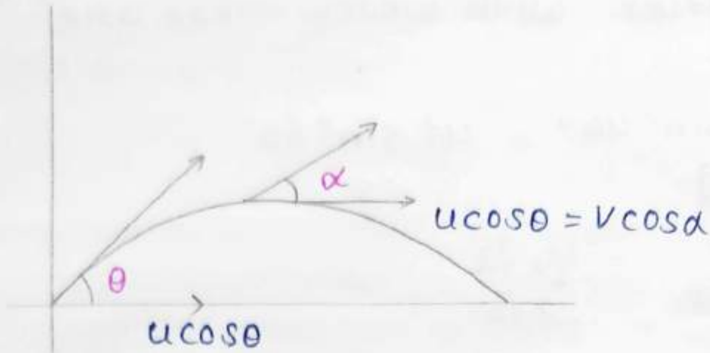
$$u_x = u\sin\theta = 10\times\sin 60^\circ$$
$$= 10\times\frac{\sqrt{3}}{2}$$
$$= 5\sqrt{3}\ ms^{-1}$$

Que- A bullet is fired from a gun at the speed of $280\ ms^{-1}$ in the direction 30° above the horizontal. The maxm height attained by the bullet is ____ ($g = 9.8\ m/s^{-2}$, $\sin 30^\circ = 0.5$)

$$H = \frac{u^2\sin^2\theta}{g} = \frac{280\times 280\times(\sin^2 30^\circ)}{2\times 10}$$

$$= \frac{280\times 14}{4} \Rightarrow 70\times 14$$
$$\Rightarrow 980\,m$$

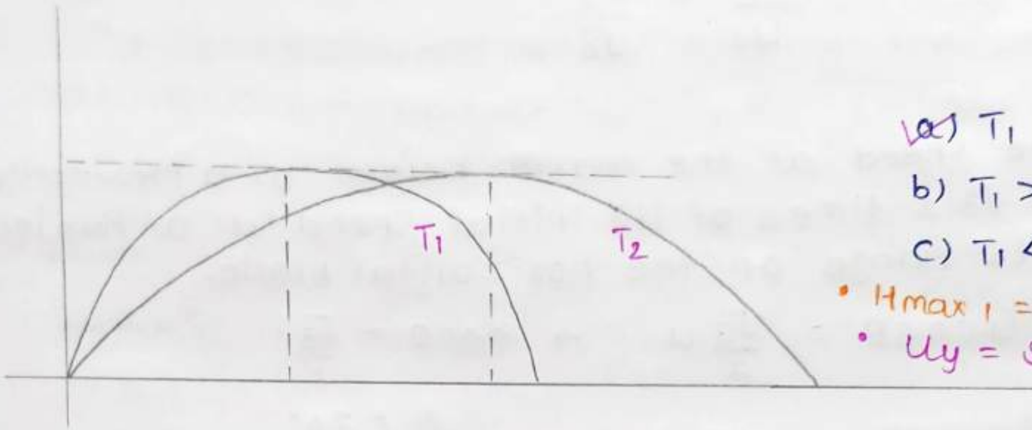
Que- A particle is thrown with the speed u at angle θ then find its speed when it is moving at an angle α from x-axis.



$$V\cos\alpha = u\cos\theta$$

$$V = \frac{u\cos\theta}{\cos\alpha}$$

$$= u\cos\theta \cdot \sec\alpha$$



(a) $T_1 = T_2$

b) $T_1 > T_2$

c) $T_1 < T_2$

- $H_{max_1} = H_{max_2}$
- u_y = same

Relation Between Horizontal Range & Maximum Height

$$H = \frac{u_y^2}{2g}$$

$$R = \frac{2u_y u_x}{g}$$

$$\rightarrow \frac{H}{R} = \frac{u_y}{4u_x}$$

$$H = \frac{R\tan\theta}{4}$$ (MR Ratta)

Que- The horizontal range & the maximum height of a projectile are equal. The angle of projection of the projectile is

$$H = \frac{R\tan\theta}{4} \quad (H = R)$$

$$\rightarrow \tan\theta = 4$$

$$\rightarrow \theta = \tan^{-1}(4)$$

Que- Two bodies are thrown up at angles of 45° and 60° respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is:

$$H_1 = H_2 \rightarrow \frac{u_1^2 \sin^2 45°}{2g} = \frac{u_2^2 \sin^2 60°}{2g}$$

$$u_1 \frac{1}{\sqrt{2}} = \frac{u_2 \sqrt{3}}{2\sqrt{2}}$$

$$\frac{u_1}{u_2} = \frac{\sqrt{3}}{\sqrt{2}}$$

Que- The speed at the maxm height of a projectile is $\sqrt{3}/2$ times of its initial speed 'u' of projection. Its range on the horizontal plane.

$$u\cos\theta = \frac{\sqrt{3}}{2}u \rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30°$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(2 \times 30°)}{g}$$

$$= \frac{\sqrt{3}\, u^2}{2g}$$

Que- An arrow is shot into the air. Its range is 200 meters and its time of flight is 5 s. If the value of g is assumed to be 10 ms-2 then the horizontal component of the velocity of arrow is

$$R = u_x \times T_f$$

$$200 = u_x \times 5$$

$$40 = u_x$$

Condition of Maximum Horizontal Range

$R_{max} = \frac{u^2 [\sin 2\theta]_{max}}{g}$

$\sin(2\theta)_{max} = 1$

$2\theta = 90^\circ$

at $\theta = 45^\circ$

$(R_{max})_{\theta=45^\circ} = \frac{u^2}{g}$

complementary angle

44°	46°
40°	50°
35°	55°
5°	85°

Que- Ball is projected at 41° and 48° then Range at 41° is R_1 and at 48° is R_2 (u = same)

(a) $R_1 = R_2$ (b) $R_1 > R_2$ ✓(c) $R_1 < R_2$

Que- A missile is fired for maxm range with an initial velocity of 20 m/s. If $g = 10\ m/s^2$, the range of the missile is.

$\theta = 45^\circ$, $u = 20$

$R = \frac{u^2}{g} = \frac{20 \times 20}{10} = 40\,m$

Que- A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection is (Home work)

Que- The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is

$$u\cos\theta = \frac{u}{2}$$

$$\cos\theta = \frac{1}{2} \rightarrow \theta = 60^\circ$$

Que- Two stones are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\pi/3$ and its maximum height is y, then the maximum height of the other will be:

$$\theta + \alpha = 90^\circ$$

$$60^\circ + \alpha = 90^\circ$$

$$\alpha = 90 - 60$$

$$= 30^\circ$$

$$\frac{Y_1 = \frac{u^2 \sin^2 60^\circ}{2g}}{Y_2 = \frac{u^2 \sin^2 30^\circ}{2g}} \rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{1}{\left(\frac{1}{2}\right)^2}$$

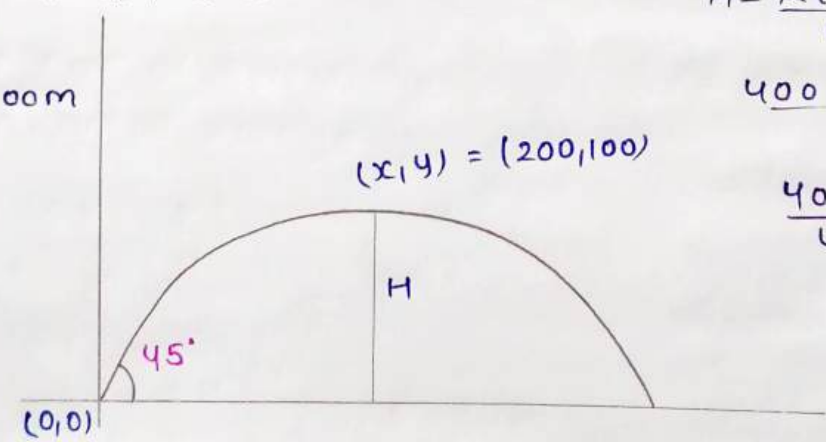
$$\frac{Y_1}{Y_2} = 3$$

$$Y_2 = \frac{Y_1}{3}$$

Que- A projectile is thrown into space so as to have the maxm possible horizontal range ~~to~~ equal to 400m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum are

$$\theta = 45^\circ$$

$$R_{max} = 400m$$



$$H = \frac{R\tan\theta}{4}$$

$$\frac{400 \times \tan 45^\circ}{4}$$

$$\frac{400}{4} = 100$$

Que- A projectile has same range at two different angle, if time of flight are t_1 and t_2 them range of the projectile will be? (Speed of projection in both the case is same)

$$t_1 = \frac{2u\sin\alpha}{g} \quad -(i)$$

$$t_2 = \frac{2u\sin\beta}{g} = \frac{2u\sin(90-\alpha)}{g}$$

$$= \frac{2u\cos\alpha}{g}$$

$$\alpha + \beta = 90^\circ$$

$$t_1 t_2 = \frac{2u\sin\alpha}{g} \times \frac{2u\cos\alpha}{g}$$

$$\frac{1}{2} g t_1 t_2 = \frac{u^2 2\sin\alpha\cos\alpha}{g}$$

$$= \frac{u^2\sin 2\alpha}{g}$$

$$\boxed{\frac{1}{2} g t_1 t_2 = R}$$

Que- Ball is projected with same speed at two different angle α and β then range is same at both angle then find Range in terms of their Height H_1 and H_2

→ u = same
→ $\alpha + \beta = 90^\circ$
→ $H_1 H_2$.

$R = 4\sqrt{H_1 H_2}$

MR*

$$H_1 = \frac{R\tan\alpha}{4} \quad -(i)$$

$$H_2 = \frac{R\tan(90-\alpha)}{4} \quad -(ii)$$

IIT Adv 2016

$$\frac{H_1}{H_2} = \frac{\tan\alpha}{\cot\alpha} = \tan^2\alpha$$

$$H_1 \times H_2 = \frac{R^2}{16}\tan\alpha\cot\alpha$$

$$R^2 = 16 H_1 H_2$$

$$R = \sqrt{16 H_1 H_2}$$

$$= 4\sqrt{H_1 H_2}$$

Equation of Trajectory

→ Relation b/w x and y

$$x = u\cos\theta\, t \quad \text{---} (I) \quad \rightarrow \quad t = \frac{x}{u\cos\theta}$$

$$y = u\sin\theta\, t - \frac{1}{2}gt^2 \quad \text{---} (II)$$

Putting the value of time in eqn (ii)

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = \tan\theta\, x - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

Eqn of Trajectory

$y = ax - bx^2$ (Parabolic Path)

$$\Rightarrow y = x\tan\theta\left[1 - \frac{gx}{2u^2\cos^2\theta\,\frac{\sin\theta}{\cos\theta}}\right]$$

$$\rightarrow y = x\tan\theta\left[1 - \frac{gx}{u^2\, 2\cos\theta\cdot\sin\theta}\right]$$

$$\rightarrow y = x\tan\theta\left[1 - \frac{x}{R}\right]$$ MR Ratta

Que- The eqn of projectile is $y = \sqrt{3}x - g\frac{x^2}{2}$, the angle of projection is:

$$y = \sqrt{3}x - g\frac{x^2}{2}$$

y

$$\cancel{x}\tan\theta = \sqrt{3}\cancel{x}$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

Que- The eqn of projectile is $y = 16x - \frac{x^2}{4}$ the horizontal range is

$$y = 16x - \frac{x^2}{4}$$

MR* $16\cancel{x} = \frac{x^{\cancel{2}}}{4}$

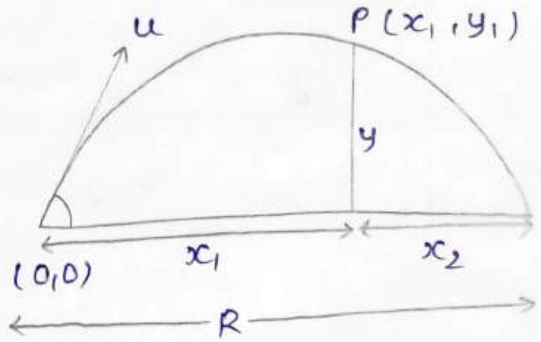
$x = 64\,m$

$$y = 16x\left(1 - \frac{x}{64}\right)$$

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$ $R = 64\,m$

Que- Find relation between x_1, x_2 and Y

Ball is projected with speed u at angle θ



$$y = x_1 \tan\theta \left(1 - \frac{x_1}{R}\right)$$

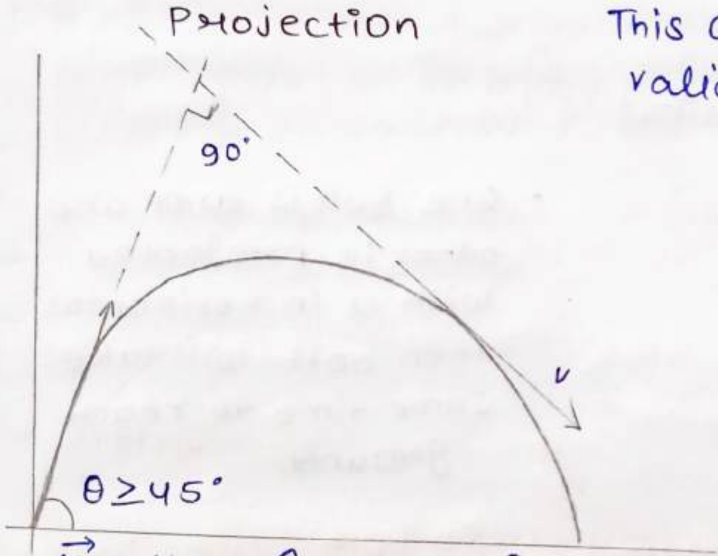
$$y = x_1 \tan\theta \left(1 - \frac{x_1}{x_1 + x_2}\right)$$

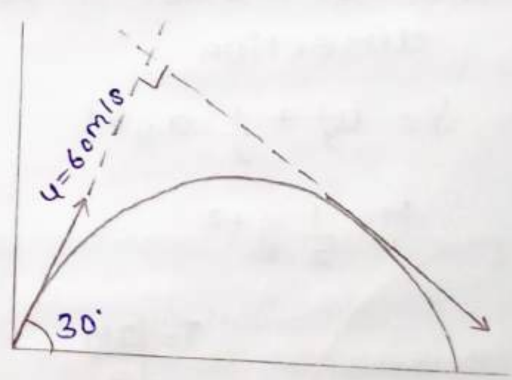
$$y = x_1 \tan\theta \left(\frac{x_1 + x_2 - x_1}{x_1 + x_2}\right)$$

$$= \frac{x_1 x_2 \tan\theta}{x_1 + x_2}$$

Que- Ball is projected with speed u at an angle θ then find time after which ball will be moving perpendicular to the initial direction of projection

This que is only valid when $\theta \geq 45°$





Not possible

$$\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$$

$$\vec{V}_f = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

$$\vec{u}_i \cdot \vec{V}_f = 0$$

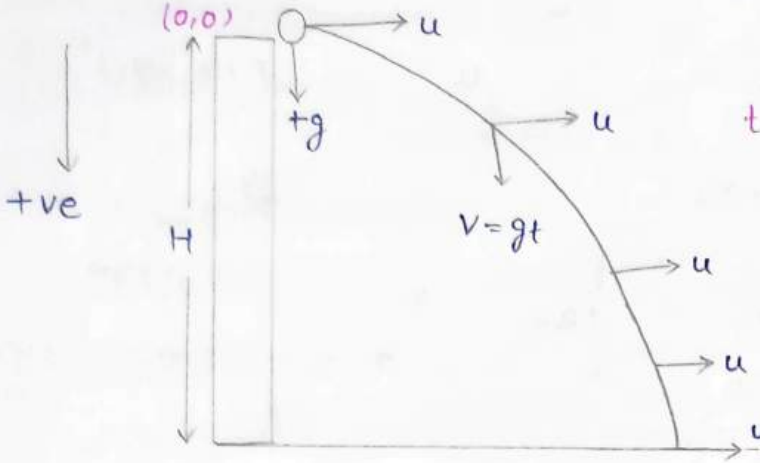
MR Ratta $\quad t = \dfrac{u}{g\sin\theta}$

$$\underbrace{u^2\cos^2\theta + u^2\sin^2\theta}_{} - u\sin\theta\, gt = 0$$

$$\cancel{u}^2 = \cancel{u}\sin\theta\, gt$$

Total angular dispm of object is projectile motion is 2θ

Que- Ball is Projected in horizontal direction with Speed u from a tower of Height 'H'



	X-axis	Y-axis
t=0	$u_x = u$	$u_y = 0$ (drop)
	$a_x = 0$	$a_y = g$
	→ uniform velocity along x	$V_y = u_y^{\,0} + a_y t$
	$V_x = u$	$V_y = gt$
	$x = ut$	$y = \frac{1}{2}gt^2$

Velocity at time 't'

$\vec{V} = u\hat{i} + gt\hat{j}$

Eqn of Trajectory → $y = \frac{1}{2}g\left(\frac{x}{u}\right)^2$

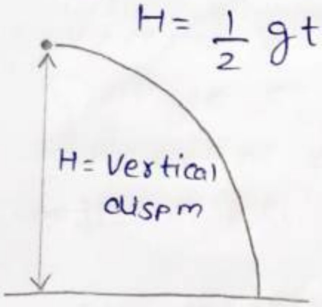
$= \frac{1}{2}g\frac{x^2}{u^2}$

Time of Flight

consider motion in vertical direction

$Y = \cancel{u_y t} + \frac{1}{2}a_y t^2$

$H = \frac{1}{2}gt^2$



$T = \sqrt{\frac{2H}{g}}$

- One Ball is drop and other is projected with u in horizontal then both will take same time to reach ground.

In both case, initial vertical velocity is zero and time of flight only depends on vertical velocity

Que- When a particle is thrown horizontally with initial velocity 'u' the resultant velocity of the projectile at any time t is given by

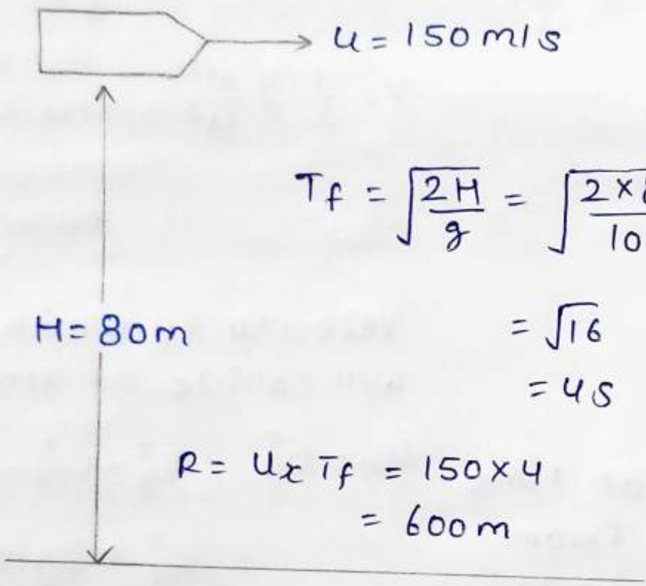
$$\vec{V} = u\hat{i} + gt\hat{j}$$

$$|\vec{V}| = \sqrt{u^2 + g^2t^2}$$

Que- A bomber is flying horizontally with a constant Speed of 150 m/s at a height of 78.4 m. The Pilot has to drop a bomb at the enemy target. At what horizontal distance from the target Should ~~be~~ he released the bomb

Acc. to $g = 9.8$ → we can take 78.48m

Acc. to $g = 10$ → we can take 80m



$$T_f = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 80}{10}}$$

$$= \sqrt{16}$$

$$= 4s$$

$$R = u_x T_f = 150 \times 4$$

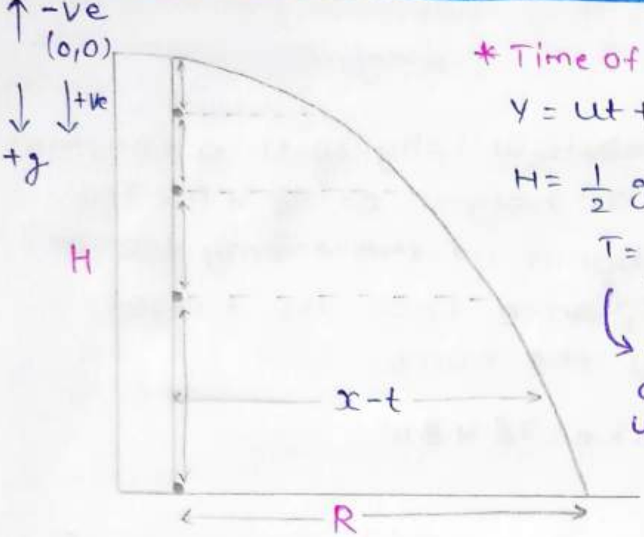
$$= 600\,m$$

Que- When a particle is thrown horizontally, with initial velocity 'u' the resultant velocity of the projectile at any time t is given by

$$\vec{V} = u\hat{i} + gt\hat{j}$$

$$|\vec{V}| = \sqrt{u^2 + g^2t^2}$$

Horizontal projectile motion



*** Time of flight**

$Y = ut + \frac{1}{2}at^2$

$H = \frac{1}{2}gt^2$

$T = \sqrt{\frac{2H}{g}}$

↳ same as drop wala case

x-axis	y-axis
$t = 0$	
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = +g$
$V_x = u$	$V_y = u + at = gt$
Uniform motion x	Variable

$\vec{V} = u\hat{i} + gt\hat{j}$

$|\vec{V}| = \text{speed} = \sqrt{u^2 + (gt)^2}$

*** Range** (consider motion in x-axis)

$x = ut$

$R = uT_f$

$R = u\sqrt{\frac{2H}{g}}$

$x = ut$ — (I) | $y = \frac{1}{2}gt^2$ — (II)

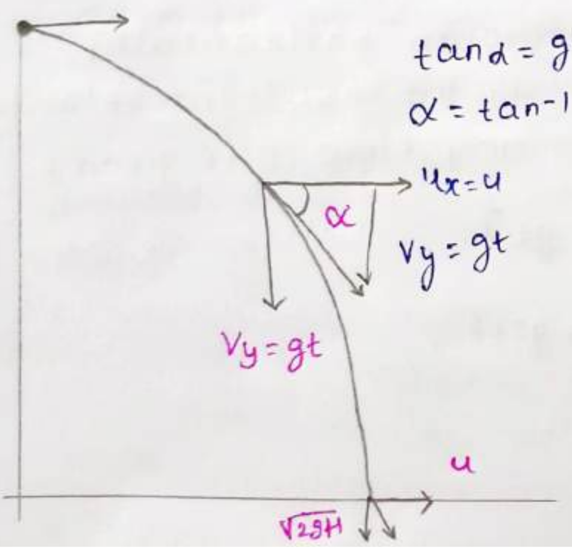
$y = \frac{1}{2}g\frac{x^2}{u^2}$ → Eqn of trajectory

Parabolic Path

Velocity at time

$\vec{V} = u\hat{i} + gt\hat{j}$

direction of motion at time 't'. Angle of velocity from Horizontal direction at 't'



$\tan\alpha = gt/u$

$\alpha = \tan^{-1}(gt/u)$

Velocity by which it will collide at ground

$u_x = u\hat{i}$ $\quad V_y^2 - u_y^2 = 2as$

$V_y^2 = 2gH$

$V_y = \sqrt{2gH}$

$\vec{V} = u\hat{i} + \sqrt{2gH}\,\hat{j}$

Angle of collision from Horizontal

$\tan\theta = \frac{\sqrt{2gH}}{u}$

$\theta = \tan^{-1}\left(\frac{u}{\sqrt{2gH}}\right)$

Que- A body is thrown horizontally with a velocity $\sqrt{2gH}$ from the top of a tower of Height h. It strikes the level ground through the foot of tower at a distance x from the tower. The value of x is

$T_f = \sqrt{\frac{2h}{g}}$ $\quad x = V_x T_f = \sqrt{2gh} \times \sqrt{\frac{2h}{g}}$

$= \sqrt{2h} \times \sqrt{2h} = 2h$

Que- Ball is projected with 30 m/s in horizontal direction from some height. Find time when it is at 45° from horizontal.

$\tan 45^\circ = \frac{gt}{u_x} \rightarrow 1 = \frac{10t}{30}$

$t = 3\ sec$

Relative Motion in Plane

(Relative motion in 1-D + vector)

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B \rightarrow$ velocity of 'A' w.r.t 'B'

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A \rightarrow$ velocity of 'B' w.r.t 'A'

$\vec{V}_{AB} = -\vec{V}_{BA}$

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

Que- velocity of Ramlal $\vec{V_R} = -3\hat{i} + 4\hat{j}$ and velocity of Pinky $\vec{V_P} = 4\hat{i} + 3\hat{j}$ then find velocity of Ramlal w.r.t Pinky

$\vec{V_R} = -3\hat{i} + 4\hat{j}$

$\vec{V_P} = 4\hat{i} + 3\hat{j}$

$V_{RP} = \vec{V_R} - (\vec{V_P})$

$= -3\hat{i} + 4\hat{j} - 4\hat{i} - 3\hat{j}$

$\vec{V}_{RP} = -7\hat{i} + \hat{j}$

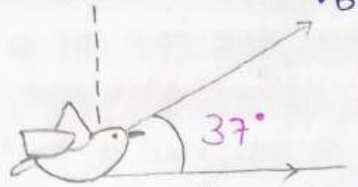
MR

$V_{Ramlal(m.R)} = 4\hat{i}$

$V_{Pinky(MR)} = 3\hat{j}$

$V_{Bird(MR)} = 4\hat{i} + 3\hat{j}$

$V_{By} = 5\sin 37 = 3\hat{j}$ $\quad V_B = 5m/s$



$$\vec{V_B} = 4\hat{i} + 3\hat{j}$$

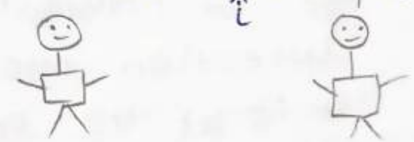
$V_{Bx} = 5 \times \cos 37°$

$5 \times \frac{4}{5} = 4m/s\ \hat{i}$

$$V_{B(Ramlal)} = \vec{V_B} - \vec{V_{RL}}$$
$$= 4\hat{i} + 3\hat{j} - 4\hat{i}$$
$$V_{B(Ramlal)} = 3\hat{j}$$

$$V_{B(pinky)} = V_B - V_P$$
$$= 4\hat{i} + 3\hat{j} - 3\hat{j}$$
$$V_{B(Pinky)} = 4\hat{i}$$

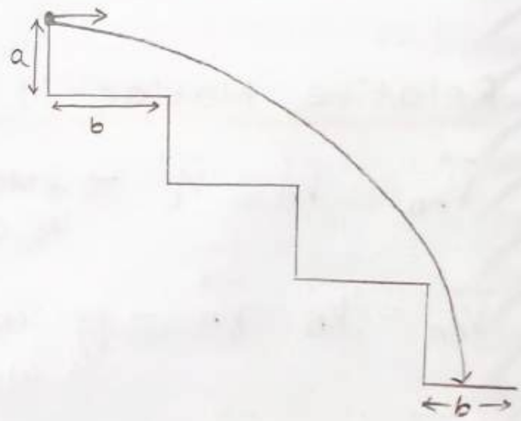
$\rightarrow V_R = 4m/s\ \hat{i}$ $\quad \uparrow V_P = 3\ m/s\ \hat{j}$



Ramlal $\quad$ Pinky

Que - Find velocity so that ball will fall on nth step

Total Height $\left(\begin{matrix} H = na \\ R = nb \end{matrix}\right)$



$$T_f = \sqrt{\frac{2(na)}{g}}$$

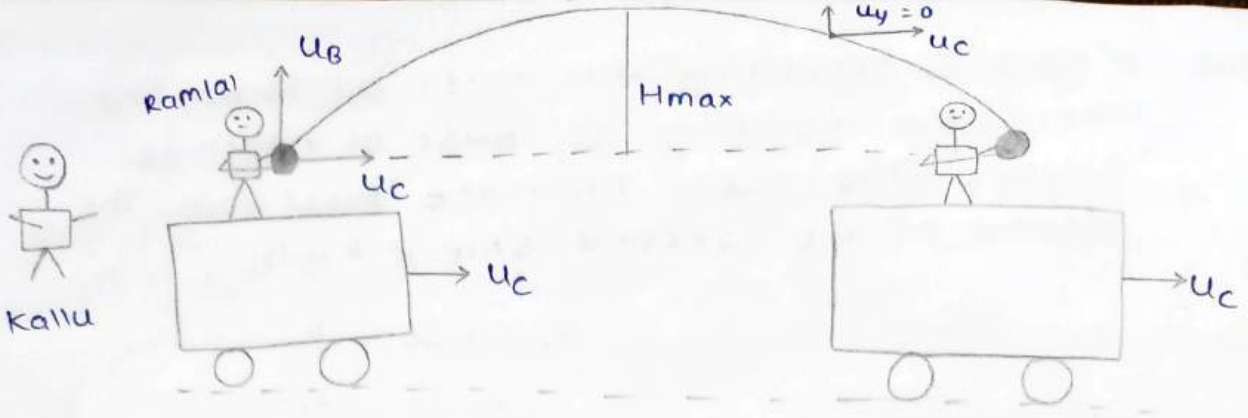
$$R = nb = uT_f$$

$$nb = u\sqrt{\frac{2na}{g}}$$

$$u = nb\sqrt{\frac{g}{2na}} = \sqrt{\frac{ngb^2}{2a}}$$

Find minm velo. so that it fall on nth step

$$u = \sqrt{\frac{(n-1)gb^2}{2a}}$$



Ball is projected with velocity u_B by man sitting on the car

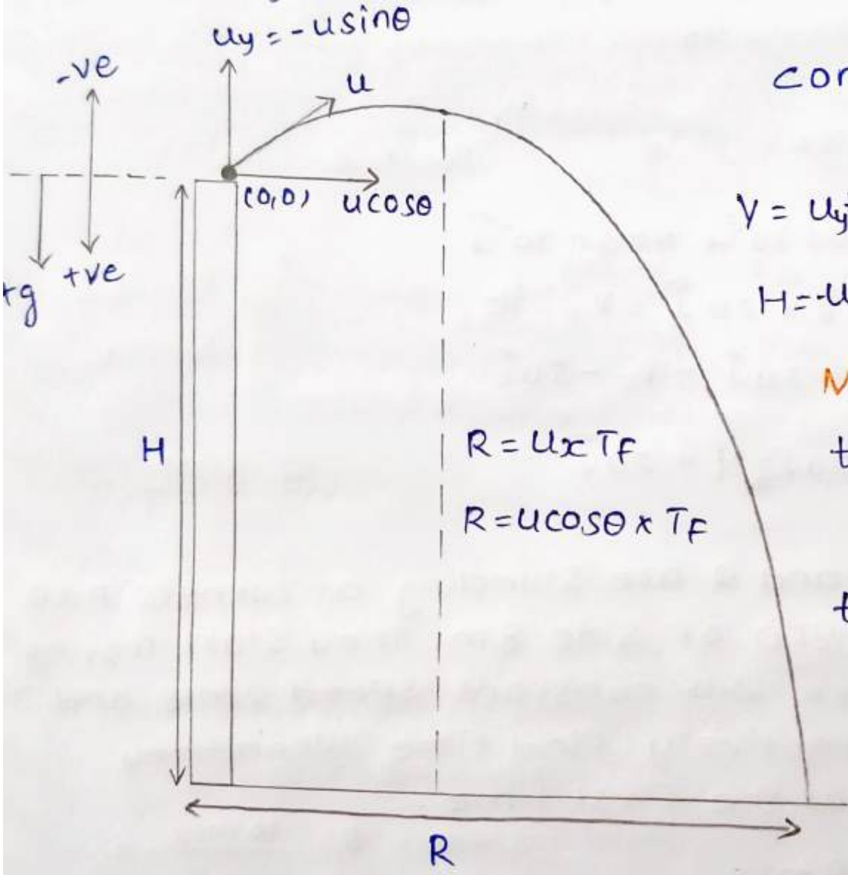
$$T_f = \frac{2u_B}{g}$$

$$R = u_x T_f$$

$$R = V_C\left(\frac{2u_B}{g}\right)$$

$$H_{max} = \frac{u^2_B}{2g}$$

- Path of Ball w.r.t Ramlal → Straight Line
- Path of Ball w.r.t Kallu → Parabolic



Consider motion along y-axis

$$y = u_y t + \frac{1}{2} a_y t^2$$

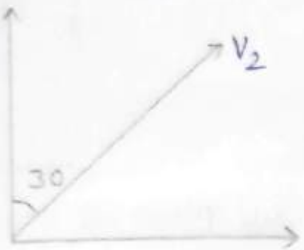
$$H = -u\sin\theta\, t + \frac{1}{2} g t^2$$

Motion under gravity

$$t = \frac{u}{g} + \sqrt{\frac{u^2}{g^2} + \frac{2H}{g}}$$

$$t = \frac{u\sin\theta}{g} + \sqrt{\frac{(u\sin\theta)^2}{g^2} + \frac{2H}{g}}$$

Que- A ship is travelling due east at 10 km/hr. Other ship heading 30° east of north is always due north from the first ship. The speed of the second ship in km/h



$V_1 = 10\,km/h\ \hat{i}$

$$V_2 \sin 30^\circ = V_1$$

$$V_2 \times \frac{1}{2} = V_1$$

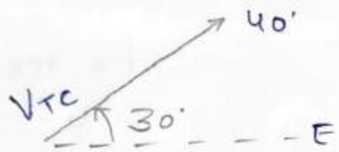
$$V_2 = 10 \times 2$$

$$= 20\ km/hr$$

Que- Car is moving with 30 m/s along east and truck is moving with speed 40 m/s at 30° N of E w.r.t car then find velocity of truck.

$V_C = 30\hat{i}$

$V_T = ?$



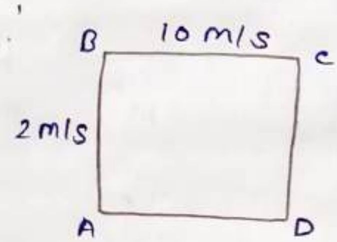
$$V_{TC} = 40\cos 30^\circ \hat{i} + 40 \sin 30^\circ \hat{j}$$

$$V_{TC} = 20\sqrt{3}\,\hat{i} + 20\,\hat{j} = V_T - V_C$$

$$20\sqrt{3}\,\hat{i} + 20\,\hat{j} = V_T - 30\hat{i}$$

$$V_T = (30 + 20\sqrt{3})\hat{i} + 20\hat{j}$$

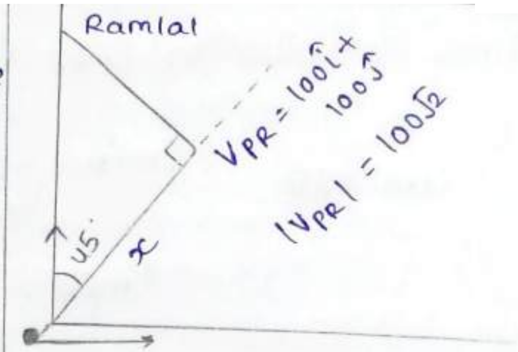
Que- Two men P and Q are standing at corners A & B of square ABCD of side 8 m. They start moving along the track with constant speed 2 m/s and 10 m/s respectively. Find time when they will meet for the first time.



$$V_{BA} = 8\ m/s$$

$$t = \frac{24}{8} = 3\ sec$$

Find minimum distance between them, min^m separation b/w Ramlal and pinki



$$\vec{V}_{PR} = \vec{V}_P - \vec{V}_R$$
$$= 100\hat{i} - (-100\hat{j})$$

$$V_{PR} = 100\hat{i} + 100\hat{j}$$

↓

Velocity of Pinki w.r.t Ramlal

↓

Assume to be at rest

$V_P = 100 m/s\ \hat{i}$

$\sin 45° = d_{min}/100$

$d_{min} = 100 \times \frac{1}{\sqrt{2}} = \frac{100}{\sqrt{2}}$

$= 50\sqrt{2}$

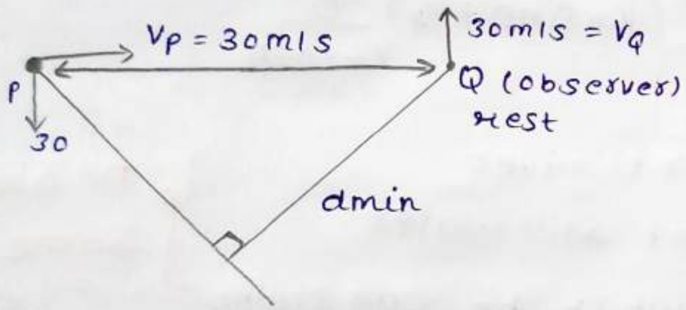
$\rightarrow \tan 45° = \frac{d_{min}}{x}$

$x = \frac{d_{min}}{\tan 45°}$

$x = \frac{100}{\sqrt{2}}$

$t = \frac{x}{u_{PR}} = \frac{100}{\sqrt{2} \times 100\sqrt{2}} = \frac{1}{2}$ sec

Que - Find min^m separation b/w them

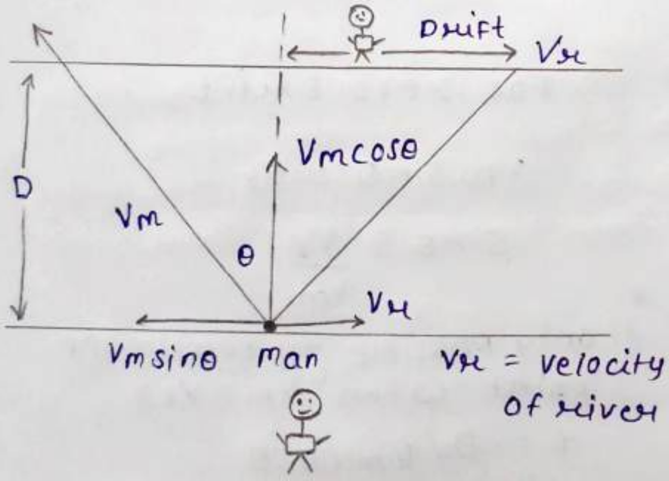


$d_{min} = \frac{80}{\sqrt{2}}$

$= 40\sqrt{2}$

River Man Problem

Cross River in min^m time



Velocity of river will support to cross the river, or oppose to cross the river

↳ Not support
Not oppose

Time to cross the river (consider motion across river)

$t = \frac{D}{u_m \cos\theta}$

$t_{min} = \frac{D}{v_m |\cos\theta|_{max}}$

$\cos\theta_{max} = 1$

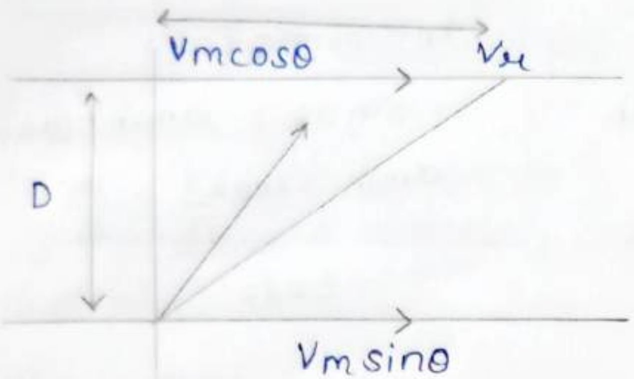
$\theta = 0°$

$t_{min} = \frac{D}{v_m}$

Drift in the case of min. time

Drift $= V_r \times t_{min}$

$= V_r \times \frac{D}{V_m}$



Time to cross the river

$t = \frac{D}{V_m \cos\theta}$

$V_{mx} = V_m \sin\theta + V_r$

↑ velocity w.r.t ground along x-axis

Drift $= (V_{mx})\, t_{crossing}$

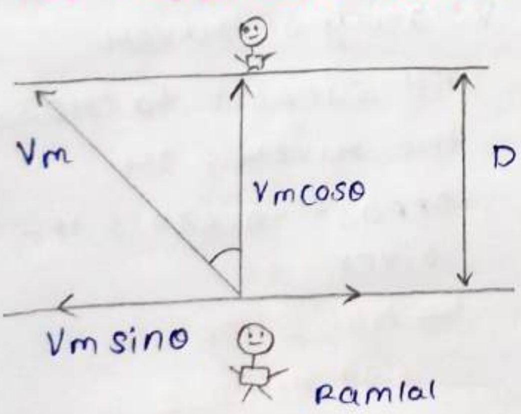
$= (V_m \sin\theta + V_r) \frac{D}{V_m \cos\theta}$

→ velocity of man w.r.t river
→ velocity of man w.r.t still water
→ velocity of man by which he can swim

} All are Same

$\vec{V}_{mr} = \vec{V}_m + \vec{u}_R$

For minimum Drift



For zero Drift

$V_m \sin\theta = V_r$

$\sin\theta = \frac{V_r}{V_m}$

(Only possible to reach opp. point when $V_m > V_r$)

$t = D / V_m \cos\theta$

Que- A river is flowing from East to west at a speed of 5m/min. A man on south bank of river, capable of swimming 10m/min in still water, wants to swim across the river in shortest time, he would swim

a) due north

b) due north-east

c) Due north-east with double the speed of river

d) none of the above

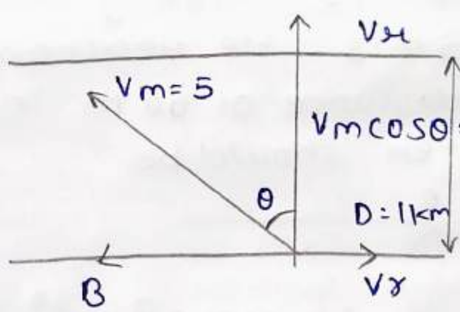
Que- A boat, which has a speed of 5km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is-

$V_B = 5\ km/h$

$V_{Br} = 3\ km/h$

$V_{Br} = V_r$

$= 3\ km/hr$



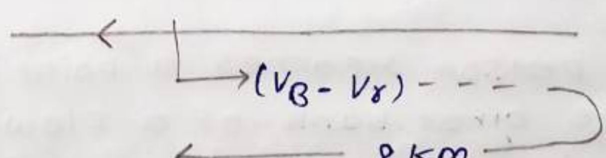
$V_m \cos\theta = \frac{D}{t} = \frac{1km}{1/4 hr}$

$= 4\ km/hr$

Que- A boat takes 2 hours to go 8 km and come back in still water lake. With water velocity of 4km/hr, the time taken for going upstream of 8 km and coming back is

$V_B = 16\ km/2hr$

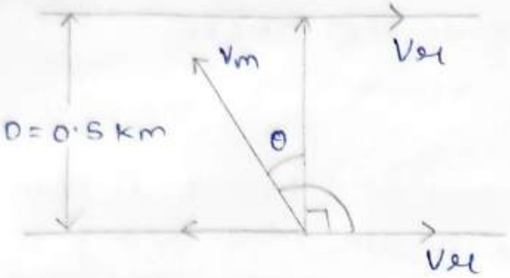
$= 8\ km/hr$



$t = \frac{8}{V_B - V_r} + \frac{8}{V_B + V_r}$

$t = \frac{8}{8-4} + \frac{8}{8+4} = 2hr + \frac{8}{12} = \frac{8}{3} hr \rightarrow \frac{8}{3} \times 60 = 160 min$

Que - A man wishes to swim a cross a river 0.5 km. wide. If he can swim at the rate of 2 km/h in still water and the river flows at the rate of 1 km/h. The angle (w.r.t the flow of the river) along which he should swim so as to reach a point exactly opposite his starting point should be



$V_m = 2$ km/hr

$V_r = 1$ km/hr

$V_m \sin\theta = V_r$

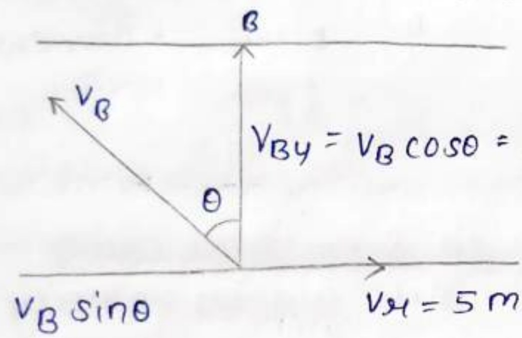
$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$\text{Angle} = 90° + 30 = 120°$

Que - A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at distance of 60 m in 5 s. His velocity in still water should be



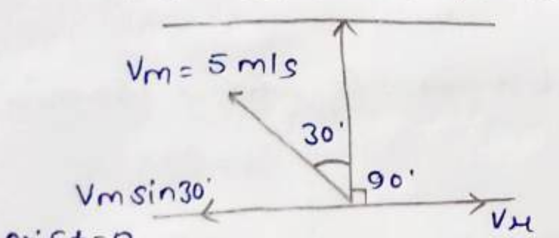
$V_{By} = V_B \cos\theta = \frac{D}{t} = \frac{60}{5} = 12$ m/s — (i)

$V_B^2 [\sin^2\theta + \cos^2\theta] = 12^2 + 5^2$

$V_B = \sqrt{169} = 13$ m/s

$V_B \sin\theta = 5$ — (ii)

Que - A person reaches a point directly opposite on the other bank of a flowing river, while swimming at a speed of 5 m/s at an angle of 120° with the flow, The speed of the flow must be

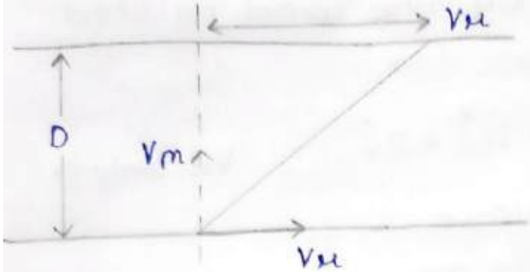


$V_m \sin 30° = V_r$

$5 \times \frac{1}{2} = V_r$

2.5 m/s $= V_r$

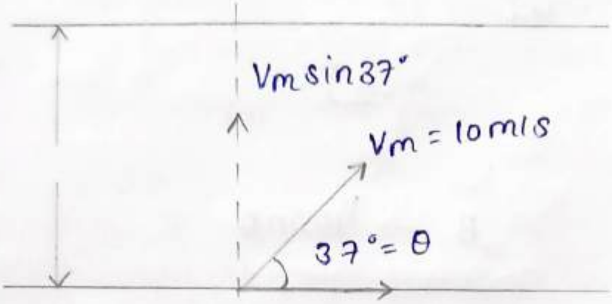
Que- River is flowing with speed 20 m/s. A man can swim in flowing river with speed 10 m/s then find drift in a case of minimum time. width of river is 60m



$$t = \frac{D}{V_m} = \frac{60}{10} = 6s$$

$$Drift = V_r t = 20 \times 6 = 120m$$

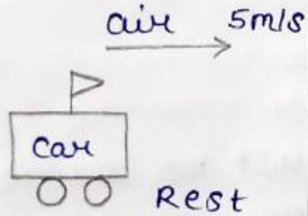
Que- Find drift and time taken to cross the river



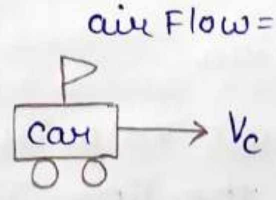
$$t = \frac{D}{V_m \sin 37°} = \frac{50}{10 \times \frac{3}{5}} = \frac{50}{6} s$$

$$Drift = V_m \cos 37° + V_r$$
$$= 10 \times \frac{4}{5} + 8$$
$$= 16 m/s$$

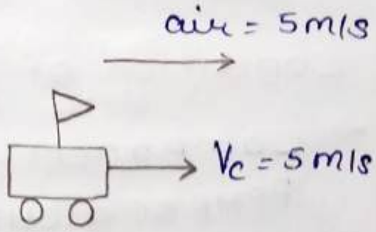
$$Drift = 16 \times \frac{50}{6} = \frac{400m}{3}$$



Flag will blown in East.



Flag will blown in west



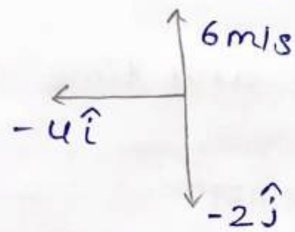
Flag will not blown

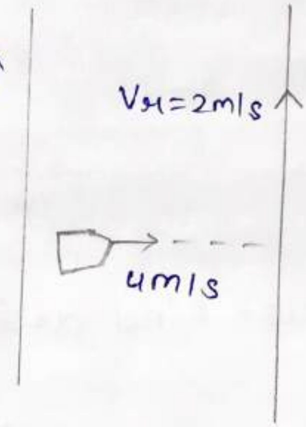
Que - A boat moving towards east with velocity 4 m/s with respect to still water and river is flowing towards north with velocity 6 m/s. The direction of the flag blown over by the wind hoisted on the boat is

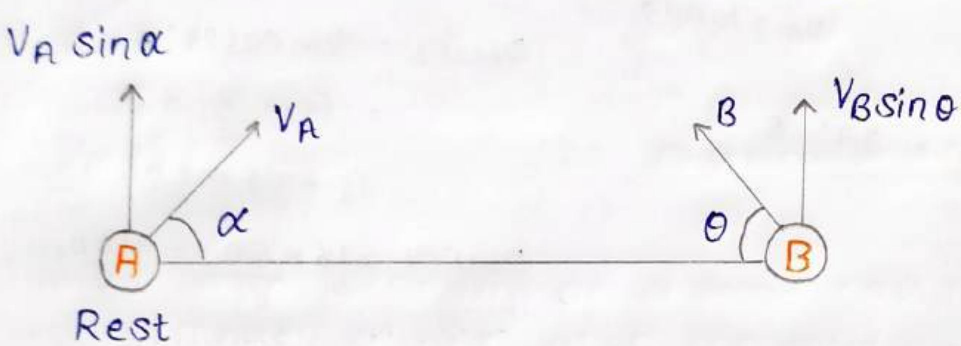
a) North-west
b) South-east
c) $\tan^{-1}(1/2$ with east)
d) North

$(V_{BG})_{net} = 4\hat{i} + 2\hat{j}$

Flag will blown opposite to me.







Condition of collision

→ Component of their velocity must be same perpendicular to the line joining.

$V_{BA} = 0$ perpendicular to line joining.

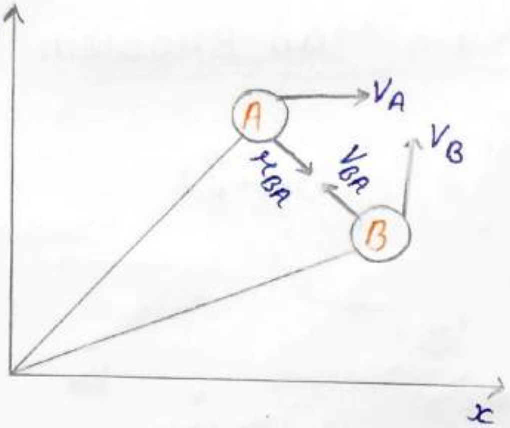
→ direction of relative velocity must be in opposite to the direction of relative position

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

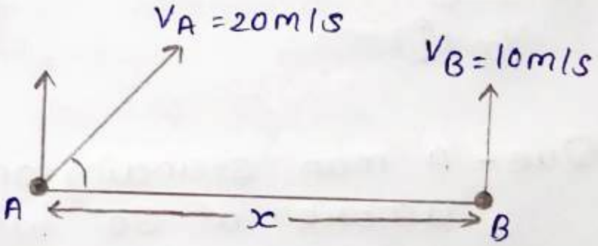
$\vec{r}_{BA} = -\vec{V}_{BA}$ (dimensionally wrong)

$\hat{r}_{BA} = -\hat{V}_{BA}$ → condition of collision

$$\frac{\vec{r}_{BA}}{|\vec{r}_B - \vec{r}_A|} = \frac{-\vec{V}_{BA}}{|\vec{V}_B - \vec{V}_A|}$$



Que- Two particles A and B are projected from the ground simultaneously in the directions shown in the figure with initial velocities $V_A = 20\,m/s$ and $V_B = 10\,m/s$ respectively. They collide after 0.5s. Find out the angle θ and the distance x



$V_A \sin\theta = V_B$

$20 \sin\theta = 10$

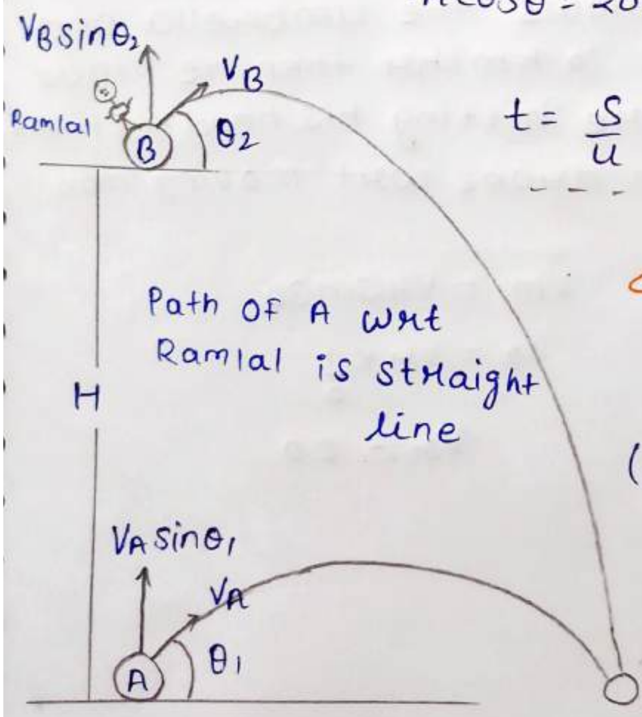
$\sin\theta = 1/2$

$\theta = 30°$

$V_A \cos\theta = 20 \times \cos 30° = \frac{20\sqrt{3}}{2}$

$t = \frac{S}{u} = \frac{x}{10\sqrt{3}} \rightarrow \frac{1}{2} = \frac{x}{10\sqrt{3}}$

$x = 5\sqrt{3}$



Cond^n of Collision

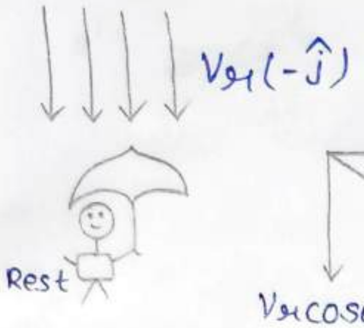
$V_B \cos\theta_2 = V_A \cos\theta_1$

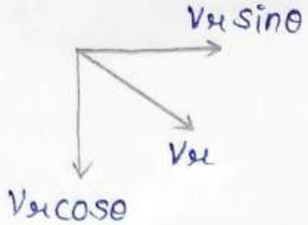
$(V_{AB})_y = V_A \sin\theta_1 - V_B \sin\theta_2$

$a_{AB} = 0 \qquad (S_{AB})_y = H$

$$t = \frac{H}{(V_A \sin\theta_1 - V_B \sin\theta_2)}$$

Rain Man Problem





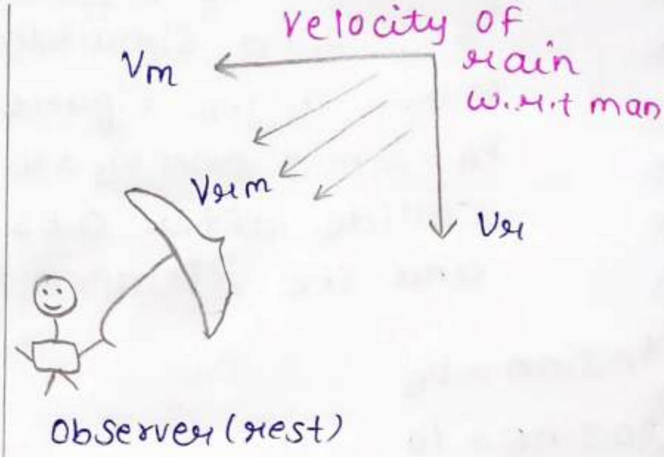
$V_{RM} = V_R - V_m$

Velocity of Rain w.r.t man

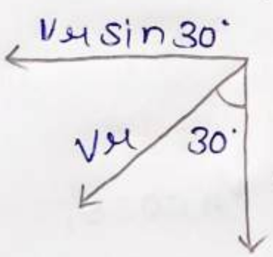
$V_{MR} = V_m - V_R$

Velocity of man w.r.t Rain

(Rain is Falling Vertically downward)



Que- A man standing on a road has to hold his umbrell at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/hr then he finds that rain drops are hitting his head vertically then speed of rain drops wrt moving man



$V_r \cos 30° = 20 \times \frac{\sqrt{3}}{2}$

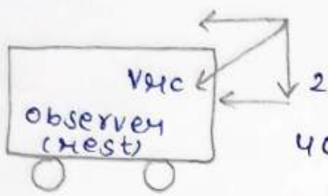
$10\sqrt{3}$

$\vec{V_m} = V_r \sin 30°$

$10 = V_r \times \frac{1}{2}$

$V_r = 20$

Que- A car with vertical winshield moves in a rain storm at a speed of 40 km/hr. The rain drops fall vertically with constant speed of 20 m/s. The angle at which rain drops strike the winshield is



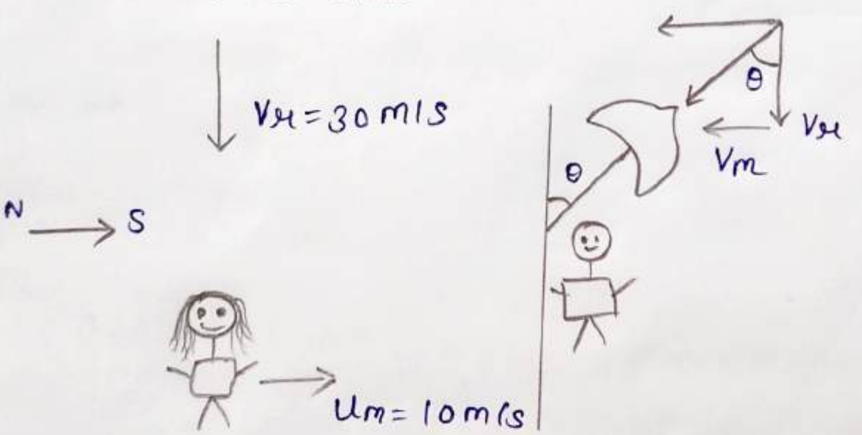
$\downarrow V_r = 20 \, m/s$

$$40 \, km/h = 40 \times \frac{5}{18}$$

$$= \frac{100}{9} \, m/s$$

$$\tan\theta = \frac{100}{9 \times 20} = \frac{5}{9} \rightarrow \theta = \tan^{-1}(5/9)$$

Que- Rain is falling vertically with a speed of $30 \, ms^{-1}$. A woman rides a bicycle with a speed of $10 \, ms^{-1}$ in the north to south direction. what is the direction in which she should hold her umbrella



$$\tan\theta = \frac{V_m}{V_r} = \frac{10}{30}$$

$$\theta = \tan^{-1}(1/3)$$

↳ Angle from vertical towards south

→ θ angle south of vertical

$$V_{rm} = -30\hat{j} - 10\hat{i}$$

$$|V_{rm}| = \sqrt{(30)^2 + (10)^2} = \sqrt{1000}$$

$$= 10\sqrt{10} \, m/s$$

Que- Two boys are standing at the end A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at ends A start running simultaneously with velocity v and catches the order in a time 't'

(1) $\frac{a}{\sqrt{v^2+v_1^2}}$

(2) $\frac{a}{v+v_1}$

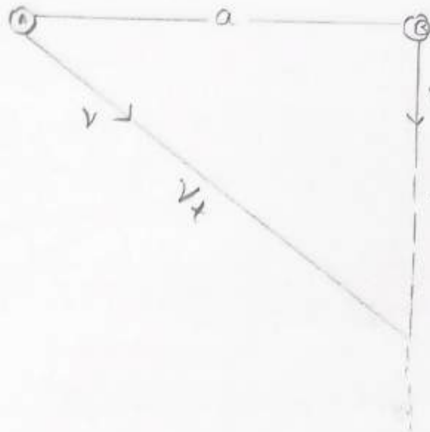
(3) $\frac{a}{v-v_1}$

(4)✓ $\sqrt{\frac{a^2}{v^2-v_1^2}}$

$$a^2 + (v_1 t)^2 = (vt)^2$$

$$a^2 = (v^2 - v_1^2)\, t^2$$

$$t = \sqrt{\frac{a^2}{v^2-v_1^2}}$$



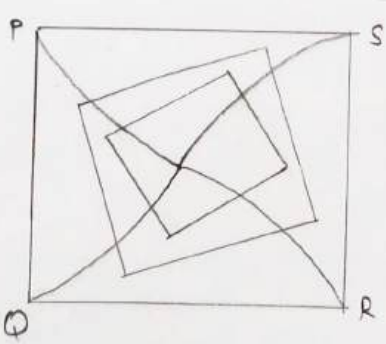
Que- four persons P, Q, R and S are initially at four corners of a square of side d. Each person now moves with a constant speed v in such a way that P always moves directly towards Q, Q towards R, R towards S, and S towards P. The four persons will meet after time

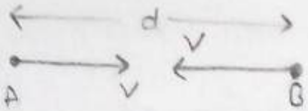
(1) $\frac{d}{2v}$

(2)✓ $\frac{d}{v}$

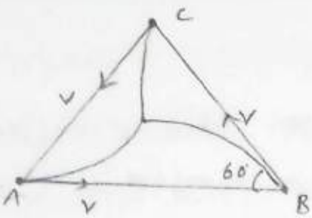
(3) $\frac{3d}{2v}$

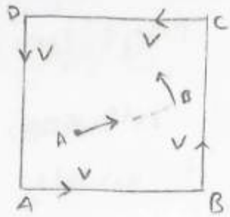
(4) They will never meet



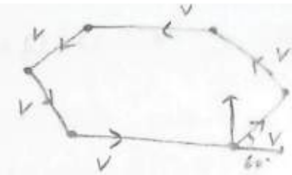


$$t = \frac{d}{2V}$$



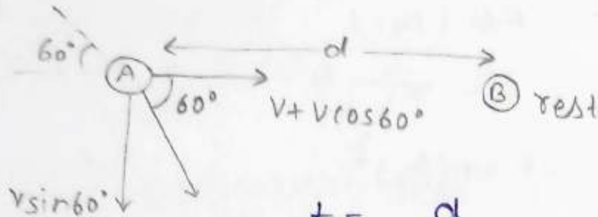


$$t = \frac{d}{V}$$



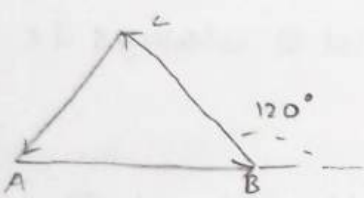
$$t = \frac{d}{V - V\cos 60^\circ}$$

$$t = \frac{d}{V - \frac{V}{2}} = \frac{2d}{V}$$



$$t = \frac{d}{V + V\cos 60^\circ} = \frac{2d}{3V}$$

n- person is standing on the corner of Polygon

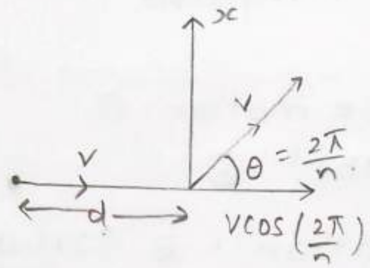


Angle b/w AB and BC = 120°

$$= \frac{360^\circ}{3} = 120^\circ$$

for square

$$\frac{360^\circ}{4} = 90^\circ$$



$$(V_{AB})_n = V - V\cos\left(\frac{2\pi}{n}\right)$$

$$t = \frac{d}{V\left[1 - \cos\left(\frac{2\pi}{n}\right)\right]}$$

n = 4 (square)

$$t = \frac{d}{V(1 - \cos 90^\circ)} = \frac{d}{V}$$

n = 3 (triangle)

$$t = \frac{d}{V(1 - \cos(120^\circ))}$$

$$= \frac{d}{V\left(1 - \left(-\frac{1}{2}\right)\right)}$$

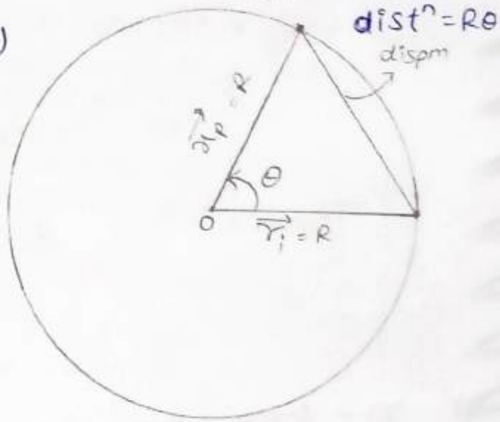
$$= \frac{2d}{3V}$$

* Circular Motion

→ When object moves on the circumference of circle then motion is called circular motion

→ Reference point is at centre of circle.

→ Dist^n moved by object = $R\theta$ (Arc)



→ Disp^m moved by object = $\vec{x_f} - \vec{x_i}$

↳ हमेशा Plane मे होगा = $2R\sin(\theta/2)$

→ Angular displacement = θ (Vector)

(Axial Vector)

→ Direction of θ ⊥r to the Plane of motion (along axis)

$\boxed{\vec{S}\cdot\theta = 0}$

S and θ always ⊥r

→ For anticlockwise motion = θ (outside the plane) ⊙

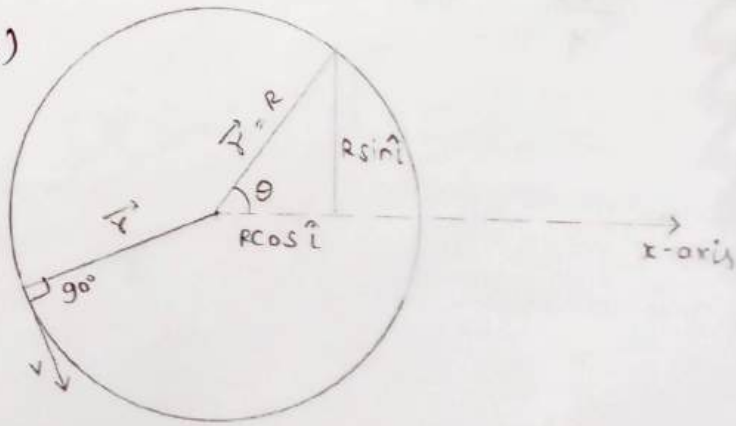
→ for clockwise motion = θ (Inward the plane) ⊗

→ Position vector along radius and away from centre.

→ velocity always perpendicular to radius along tangent. $\boxed{\vec{r}\cdot\vec{v} = 0}$

→ Axial vector All angular Parameter (θ, ω, α)

Circular Motion (2D)



→ Position of object w.r.t. centre at any time 't' is $\vec{r} = R\sin\theta\,\hat{j} + R\cos\theta\,\hat{i}$

$\vec{r}\cdot\vec{v} = (R\sin\theta\,\hat{j} + R\cos\theta\,\hat{i})\cdot(R\cos\theta\,\hat{j} - R\sin\theta\,\hat{i})$

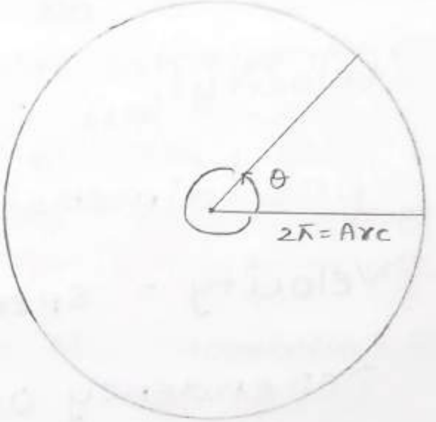
→ $R^2\sin\theta\cdot\cos\theta - R^2\cos\theta\cdot\sin\theta$

$\boxed{\vec{r}\cdot\vec{v} = 0}$

$\vec{v} = \dfrac{d\vec{R}}{dt}$

$\boxed{= R\cos\theta\,\hat{j} - R\sin\theta\,\hat{i}}$

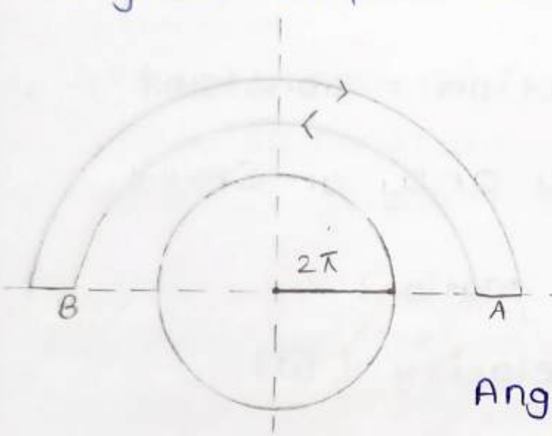
Que- find angular and linear displacement in one rotation



Distance $= 2\pi r$

Linear Dispm $= 0$

Angular Dispm $= 2\pi\ \odot$



Angular dispm $= \pi\ \odot$

Angular dispm $(A \to B - A)$

$= \pi\odot + \pi\otimes = 0$

Que- find angular dispm in π- Rotation

(a) π

(b) 2π

(c) $2\pi^2$ ✓

(d) π^2

Angular dispm in one rotation $= 2\pi$

in two rotation $= 4\pi$

in three rotation $= 6\pi$

n rotation $= n(2\pi)$

no. of Rotation is π

$\theta = \pi(2\pi)$

$= 2\pi^2$

Que- find angular dispm of hr. hand, minute hand, and sec. hand of clock in 12 hr.

θ (hour hand) = $2\pi r$ (in 12 hr)

θ (minute hand) = $12(2\pi)$ = 24π rad

θ (sec hand) = 12×60 = 2π rad

* one-D Motion

Velocity = $\frac{d\vec{x}}{dt}$

(velocity)$_{inst}$ = speed × direction

→ one dimension straight line motion

Velocity = speed × direction = constant

Dependency of velocity only on speed

* Angular speed (circular motion)

→ Magnitude of Angular velocity ($\vec{\omega}$)

\# ω → Anti clock ⊙ outward

\# ω → clock wise ⊗ Inward

Angular velocity

Avg. Angular velocity

$$\frac{\text{Total Angular dispm}}{\text{Total time taken}}$$

Instantaneous Angular velocity

How fast angle is changing

Avg. Angular velocity

$$\omega_{Avg} = \frac{\vec{\theta}_f - \vec{\theta}_i}{\Delta t}$$

$$\vec{\omega}_{Avg} = \frac{\int \omega dt}{\int dt}$$

Inst. velocity

$\vec{\omega} = \frac{d\theta}{dt}$ = slope of θ / t graph is 'ω'

unit = rad / sec

Dim^n = T^{-1}

* Uniform Angular Velo. :- The rate of change is Angle = constant

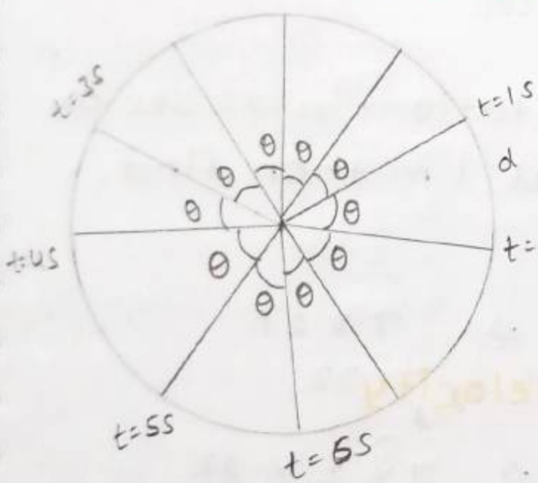
↳ same angle travelled in same time

$$\omega_{Avg} = \frac{\Delta\theta}{\Delta T}$$

ω = uniform

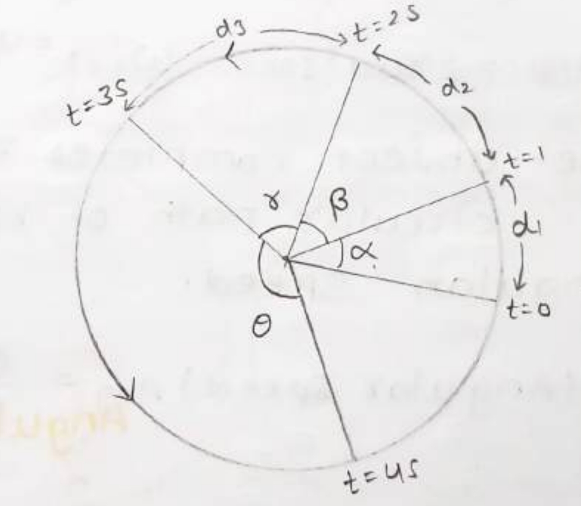
(Dist^n) Arc length is also constant

speed = cost^n



ω = non-uniform = Diff^n Angle in same time interval.

Speed → Non-uniform



Que- Find Angle moved by Angular Velocity vector in one complete rotation

(a) 2π

(b) π

(c) zero

(d) can't say

→ Angular dispm (Angle moved by object) $= 2\pi$

Que- Angular dispm of object $\theta = t^2 + 2t + 5$ then, find angular speed.

$$\omega = \frac{d\theta}{dt} = 2t + 2$$

$$\omega = 2t + 2$$

$\omega_{t=0} = 2$ rad/sec $\qquad \omega_{t=1} = 4$ rad/sec $\qquad \omega_{t=2 sec} = 6$ rad/sec

Que- object completes 7 rotation in 22 sec on circular path of radius 1 m then find angular speed.

$$(\text{Angular Speed})_{Avg} = \frac{\theta}{T} \Rightarrow \frac{7 \times 2\pi}{22}$$

$$\Rightarrow \frac{\cancel{7} \times 2 \times \cancel{22}}{\cancel{22} \times \cancel{7}}$$

$$\Rightarrow 2 \text{ rad/sec}$$

* Angular Acceleration

α

In time 't' | At time 't'

$\vec{\alpha}_{Avg} = \dfrac{\text{Total change in Angular velocity}}{\text{Total time}}$

$\vec{\alpha}_{Avg} = \dfrac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t} = \dfrac{\Delta \omega}{\Delta t}$

$\vec{\alpha}_{Avg} = \dfrac{\int \alpha \, dt}{\int dt}$

$\vec{\alpha}_{Inst} = \dfrac{d\vec{\omega}}{dt}$ » The rate of change in angular velocity is α

$\vec{\alpha} = \dfrac{d}{dt}\left(\dfrac{d\theta}{dt}\right) = \dfrac{d^2\theta}{dt^2}$

$$\left[\vec{\alpha} = \frac{d\omega}{dt} \times \frac{d\theta}{d\theta} = \frac{\omega \, d\omega}{d\theta}\right]$$

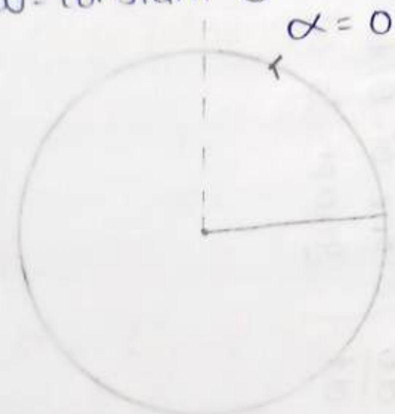
$\alpha_{Avg} = \dfrac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$ → Unit - rad/sec²

$\alpha_{Inst} = \dfrac{d\omega}{dt}$ → Dimⁿ - T^{-2}

$$\boxed{\vec{\alpha} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}}$$

ω = constant ⊙

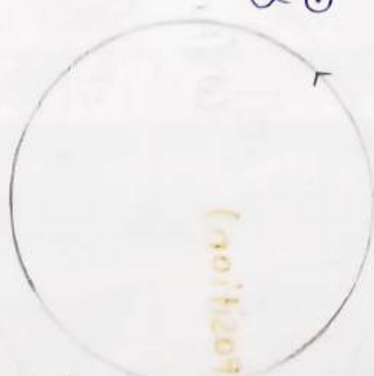
$\alpha = 0$



$\alpha = \dfrac{d\omega}{dt}$

$\omega \uparrow$ ⊙

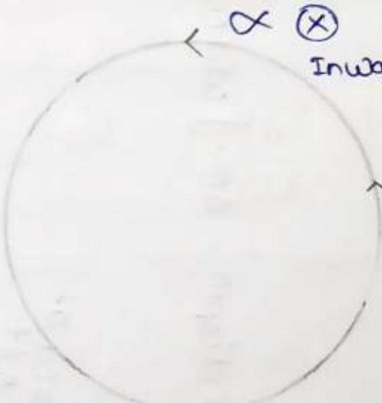
α ⊙



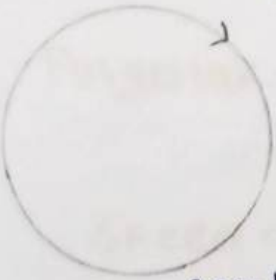
$\vec{\omega}$ & $\vec{\alpha}$ must be parallel

$\omega \downarrow$ ⊙

α ⊗ Inward



$\vec{\omega}$ & $\vec{\alpha}$ must be anti-parallel



ω ⊗

α ⊙

Angular Speed ↓

ω ⊗

α ⊙

Angular speed ↑

$\boxed{\theta, \omega, \alpha}$

→ axial vector

→ along axis

Angular dispm = $\Delta\theta = \int \omega d\theta$ = Area of ω-t graph

$\Delta\omega = \int_{t_1}^{t_2} \alpha dt$ = Area of α-t graph = $\Delta\omega$

नहीं = $\int \theta dt$ = Area of θ/t

θ (Angular Position) ω (Angular velocity) α (Angular accn)

$\omega = \frac{d\theta}{dt}$ (Slope of θ/t graph is ω)

$\alpha = \omega\frac{d\omega}{d\theta}$

$\alpha = \frac{d\omega}{dt}$ (Slope of ω/t is α)

नहीं $\frac{d\alpha}{dt}$ (slope)

* Relation b/w Linear speed and Angular speed

$s = \text{dist}^n = R\theta$

Diff^n w.r.t time

$\frac{ds}{dt} = R\frac{d\theta}{dt}$

(speed) = $R\omega$ (Angular speed)

Diff^n w.r.t time

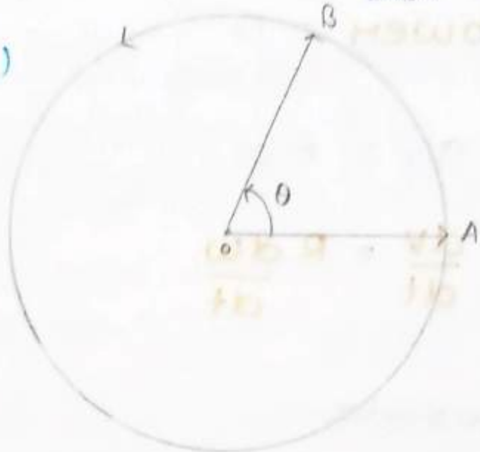
$\frac{d(\text{speed})}{dt} = R\frac{d\omega}{dt}$

$\boxed{a_T = R\alpha}$

$\boxed{a_T = R\alpha}$

Tangential acc^n

Dist^n = $R\theta$



Que- object is moving on circular path find acc^n

→ you have to find acceleration

Acceleration	Tangential acc^n	centripetal acc^n	Angular acc^n
$\vec{a} = \frac{d\vec{v}}{dt}$	$\vec{a}_T = \frac{d\lvert\vec{v}\rvert}{dt}$	$\vec{a}_c$	$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

$\vec{a} = \vec{a}_T + \vec{a}_c$ $\boxed{a_T = R\alpha}$

* Uniform circular Acceleration

Angular speed = cost^n	Angular ace^n $\left(\alpha = \frac{d\omega}{dt}\right) = 0$
Speed = constant	Tangential acc^n = 0

Velocity = changing $\quad$ $acc^n = \vec{a}_T + \vec{a}_c \neq 0$

Direction = changing $\quad$ $acc^n \neq 0$

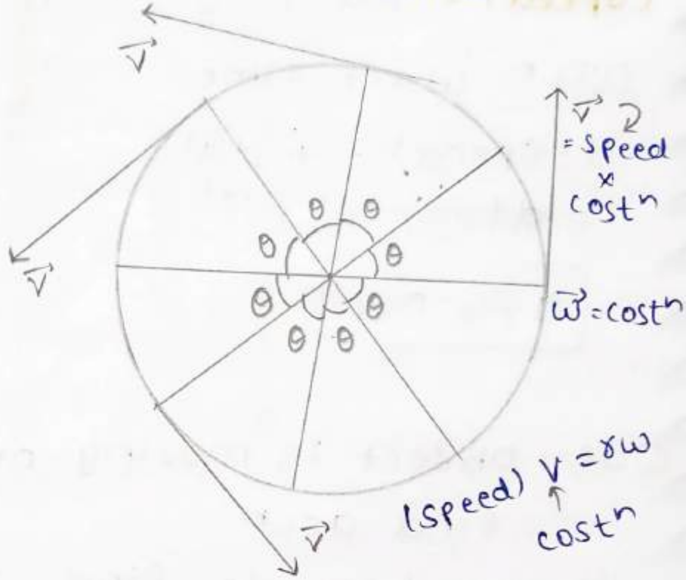
Kinetic energy = constant $\quad$ Work = $\Delta K.E = 0$

Momentum = $(\vec{P} = m\vec{v})$ = Variable $\quad$ force $\neq 0$

Power = 0 $\quad$ Power = $\frac{work}{time} = 0$

$a_T = R\alpha$

$\frac{dv}{dt} = R\frac{d\omega}{dt}$



Constant acc^n $(\vec{a})$

$\vec{v} = \vec{u} + \vec{a}t$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$v^2 - u^2 = 2as$

$S_{n^{th}} = u + \frac{a}{2}(2n-1)$

$v_{avg} = \frac{\vec{u} + \vec{v}}{2}$ $\quad$ $u_{mid} = \sqrt{\frac{u^2+v^2}{2}}$

$\boxed{S = \frac{u^2}{2a}}$

Constant Angular acc^n

$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}t$

Q- Angular speed of a uniformly circulating body with time period T is

(a) $2\pi T$

✓(b) $2\pi/T$

(c) πT

(d) π/T

$$\omega_{Avg} = \frac{\text{Total angle}}{\text{Total time}} = \frac{2\pi}{T} = 2\pi f$$

$$\boxed{f = \frac{1}{T}}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

→ Angular speed - rad/sec

→ Angular frequency.

$f \to Hz$

$T \to Sec$

Que- Angular velocity of minute hand of a clock is

(a) $\frac{2\pi}{1800}$ rad/s

(b) 8π rad/s

✓(c) $\frac{\pi}{1800}$ rad/s

(d) $\frac{\pi}{30}$ rad/s

$$\omega_{mint} = \frac{2\pi}{60\,min} = \frac{2\pi \text{ rad}}{60 \times 60 \text{ sec}}$$

$$\boxed{= \frac{\pi}{1800} \text{ rad/sec}}$$

Que- The angular velocity of the second's needle in watch is

✓(a) $\pi/30$

(b) 2π

(c) π

(d) $\frac{60}{\pi}$

$$\omega_{sec} = \frac{2\pi}{60\,sec}$$

$$= \frac{\pi}{30} \text{ rad/sec}$$

Que- The angular speed of a fly wheel making 120 revolutions/minute is

✓(a) 4π rad/s

(b) $4\pi^2$ rad/s

(c) π rad/s

(d) 2π rad/s

$\omega = 2\pi f$

$= 2\pi(2)$

$\omega = 4\pi$ rad/s

120 rot^n/min

$f = \frac{120 \text{ rot}^n}{60 \text{ sec}}$

$= 2$ rot^n/sec

Que- speed of an object moving in circular Path of radius 10 m with angular speed 2 rad/s is

(a) 10 m/s

(b) 5 m/s

✓(c) 20 m/s

(d) 30 m/s

R = 10 m

ω = 2 rad/sec

$V = r\omega$

$V = 10 \times 2$

$V = 20$ m/s

Que- A body is whirled in a horizontal ~~cycle~~ circle of radius 20 cm. It has an angular velocity of 10 rad/s. what is its linear velocity at any point on circular Path?

(a) 20 m/s

(b) $\sqrt{2}$ m/s

(c) 10 m/s

✓(d) 2 m/s

R = 20 cm

ω = 10 rad/sec

$V = \frac{20}{100} \times 10$

$V = 2$ m/s

* Uniform Circular Motion

→ It is a motion in which velocity is non-uniform with non-uniform acceleration.

Avg. velocity :- $\dfrac{\vec{r_f} - \vec{r_i}}{\Delta t} = \dfrac{2R\sin(\theta/2)}{R\theta/v}$

$$\boxed{\text{Average velocity} = (v) \times \frac{\sin\theta/2}{\theta/2}}$$

Avg. accⁿ : $\dfrac{\vec{V_f} - \vec{V_i}}{\Delta t} = \dfrac{2v\sin(\theta/2)}{R\theta/v}$

$$\boxed{(acc^n)_{avg} = \frac{v^2\sin(\theta/2)}{R\,(\theta/2)}}$$ *

$$\boxed{V_{Avg} = \frac{V\sin(\theta/2)}{\theta/2}} \qquad \boxed{\vec{\Delta V} = 2v\sin(\theta/2)}$$

$(acc^n)_{avg} = \dfrac{V^2}{R}\,\dfrac{\sin(\theta/2)}{\theta/2}$

U.C.M.

$\vec{a} = \vec{a_T} + \vec{a_c}$

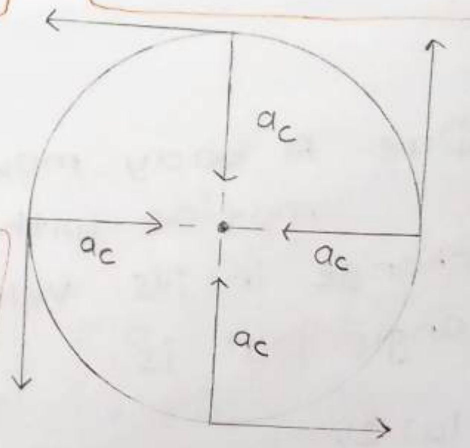
$$\boxed{\vec{a} = \vec{a_c} = \left(\frac{V^2}{R}\right)}$$

$\vec{a}_{inst} = \dfrac{V^2}{R}$

$\theta \to 0$

Centripetal accⁿ

Constant in Magnitude but dirⁿ is variable



→ along radius towards centre

Uniform Circular Motion

$$|\vec{a}| = |\vec{a_c}| = \frac{v^2}{R} = \frac{R^2\omega^2}{R} = \omega^2 R = (2\pi f)^2 R = \left(\frac{2\pi}{T}\right)^2 R$$

$$\boxed{|\vec{a}| = |\vec{a_c}| = \frac{v^2}{R} = \frac{\omega^2}{R} = 4\pi^2 f^2 R = \frac{4\pi^2}{T^2} R}$$

centripetal accn at an instant

$$\frac{\sin \theta/2}{\theta/2}$$

Que- A particle is moving in a circle of radius r having centre at O, with a constant speed v. The magnitude of change in velocity in moving from A to B is

(a) 2v

(b) 0

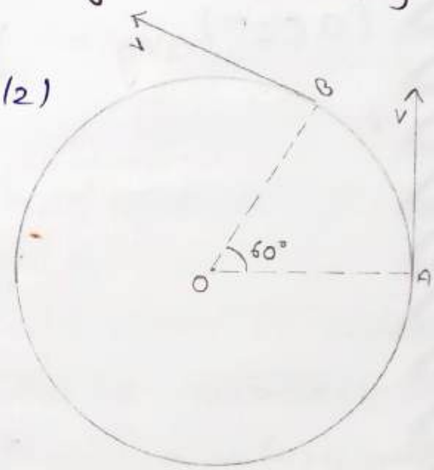
(c) $\sqrt{3}v$

(d) v

$$\Delta\vec{V} = \vec{V_f} - \vec{V_i} = 2v \sin(\theta/2)$$

$$= 2v \sin(60/2)$$

$$= 2v \times \frac{1}{2} = v$$



Que- A body moving in a uniform circular motion with speed v, the magnitude of change in its velocity after it rotates by an angle 120° is

(a) 2v

(b) $\sqrt{3}v$

(c) v

(d) $\frac{v\sqrt{3}}{2}$

$$\vec{\Delta V} = 2v \sin(\theta/2)$$

$$= 2v \cdot \sin(120°/2)$$

$$= \sqrt{3}v$$

Que- What would be the distance - time graph of uniform circular motion ?

(a) curved line

(b) Straight line

(c) semicircle

(d) None

Speed = constant = $\frac{\text{distance}}{\text{time}}$

Distance = Speed x time

Que- If a particle is moving in a uniform circular motion ________

(a) No tangential acceleration is seen.

(b) speed is constant.

(c) velocity is changing at every instant.

(d) All the above options

Que- An object moving in a circular path at constant speed has constant

(a) Energy (Kinetic Energy = Constant)

(b) velocity

(c) Acceleration

(d) Displacement

Que- In uniform circular motion

(a) acc^n is variable.

(b) acc^n is uniform.

(c) the direction and magnitude of acc^n both vary.

(d) if force applied is doubled in circular motion, then angular velo. become double.

$\vec{a} = \omega^2 R$

$F = ma = \omega^2 R$

$\boxed{\omega^2 \propto F}$

$\omega \propto \sqrt{F}$

Que- The magnitude of the tangential acc^n, the Particle moving in a circle of radius 10 cm with uniform Speed completing the circle in 4s will be

(a) $5\pi\ cm/s^2$

(b) $2.5\pi\ cm/s^2$

(c) $5\pi^2\ cm/s^2$

(d) zero

Uniform circular motion

$a_T = 0$

$a = a_c = \omega^2 R$

Que- A body revolves with constant speed v in a circular path of radius r. The magnitude of its average acceleration during motion between two points in diametrically opposite direction is

(a) zero

(b) $\frac{v^2}{r}$

(c) $\frac{2v^2}{\pi r}$

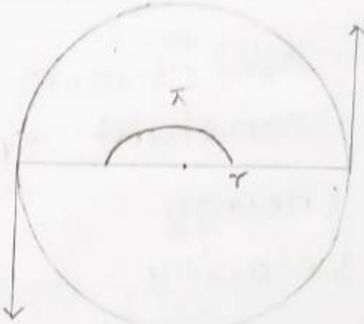
(d) $\frac{v^2}{2r}$

U.C.M.

$$\text{Avg. Acc}^n = \frac{\vec{v_f} - \vec{v_i}}{\Delta t}$$

$$= \frac{v-(-v)}{\pi R/v}$$

$$= \frac{2v^2}{\pi R}$$



$$(Acc^n)_{Avg} = \frac{v^2}{R} \frac{\sin(\theta/2)}{(\theta/2)}$$

$$= \frac{v^2}{R} \frac{\sin(\pi/2)}{\pi/2} = \frac{2v^2}{\pi R}$$

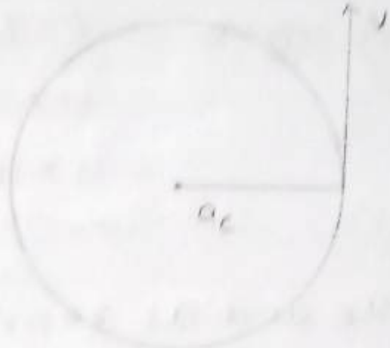
Que- The angle between velocity vector and acceleration vector in uniform circular motion is

(a) 0°

(b) 180°

✓(c) 90°

(d) 45°



Que- A car is going round a circle of radius R_1 with constant speed. Another car is going round a circle of radius R_2 with constant speed. If both of them take same time to complete the circles, the ratio of their angular speeds and linear speeds will be

(a) $\sqrt{\frac{R_1}{R_2}}, \frac{R_1}{R_2}$

(b) 1, 1

✓(c) 1, $\frac{R_1}{R_2}$

(d) $\frac{R_1}{R_2}$

$\omega = \frac{2\pi}{T}$ = Same

$V = r\omega \rightarrow$ same

$\frac{V_1}{V_2} = \frac{R_1}{R_2}$

Que- centripetal acceleration of a cyclist completing 7 rounds in a minute along a circular track of radius 5 m with a constant speed is

✓(a) 2.7 m/s²

(b) 4 m/s²

(c) 3.78 m/s²

(d) 6 m/s²

$f = \frac{7 \text{ rotation}}{\text{minute}}$

$f = \frac{7}{60}$ Hz

$\pi = 3.14$

$\pi^2 = 10$

$$a_c = \omega^2 R = (2\pi f)^2 R$$
$$= 4\pi^2 f^2 R$$
$$= 4 \times 10 \times \frac{49}{60 \times 60} \times 5 = \frac{49}{18} = 2.7\ m/s^2$$

Que- a_r and a_t represent radial and tangential acceleration. The motion of a particle will be uniform circular motion if:-

(a) $a_r = 0$ and $a_t = 0$

(b) $a_r = 0$ but $a_t \neq 0$

(c) $a_r \neq 0$ but $a_t = 0$

(d) $a_r \neq 0$ and $a_t \neq 0$

$a_r = a_c = v^2/R$

$a_t = 0$

Que- Which of the following correct for U.C.M.

(a) $\vec{a} \cdot \vec{v} = +ve$

(b) $\vec{a} \cdot \vec{v} = -ve$

(c) $\vec{a} \cdot \vec{v} = 0$

(d) None

$\vec{a} \perp v$

Que- If the frequency of an object in uniform circular motion is doubled, its acceleration becomes

(a) Two times

(b) four times

(c) Half

(d) one fourth

$\vec{a_c} = \vec{a} = (2\pi f)^2 R$

$a_c \propto f^2$

Que- If speed of an object revolving in a circular path is doubled, then angular speed is reduced to half of original value then centripetal acceleration will become / remain

(a) same
(b) Double
(c) Half
(d) Quadruple

$a_c = \frac{v^2}{R}$, $v = R\omega \rightarrow R = \frac{v}{\omega}$

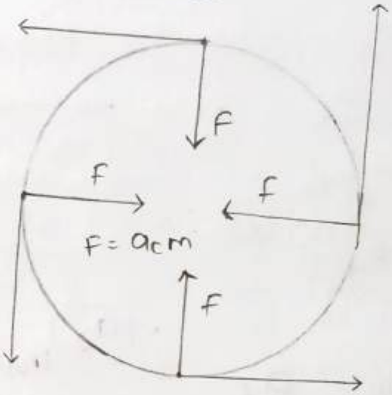
$a_c = \frac{v^2}{\cancel{v}} \times \omega \Rightarrow a_c = v\omega$

Que- A Particle is acted upon by a force of constant magnitude which is always Perpendicular to the velocity of Particle, the motion of particle takes place in a plane. It follows that

(a) its velocity is constant
(b) its accn is constant
(c) its kinetic energy is constant (speed = constn)
(d) It moves in a straight line.

Uniform ~~accn~~ circular motion

$\vec{F} = m\vec{a}_c$



* Non-Uniform circular Motion

→ $\vec{\omega}$ (Angular velocity) = variable
(Angular speed) = variable

→ $\alpha \neq 0 = \frac{d\omega}{dt}$

→ Linear speed ($v = R\omega$) = variable

$a_T \neq 0$

Direction :- variable

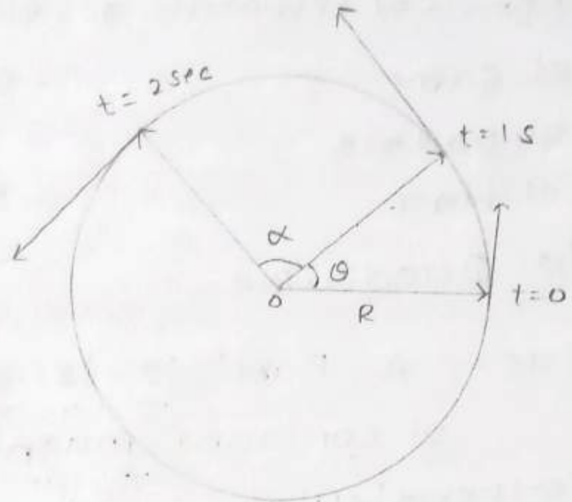
$\vec{a}_c \neq 0$

Kinetic energy = variable

Momentum = variable

Work = $\Delta K.E \neq 0$

Power $\neq 0$

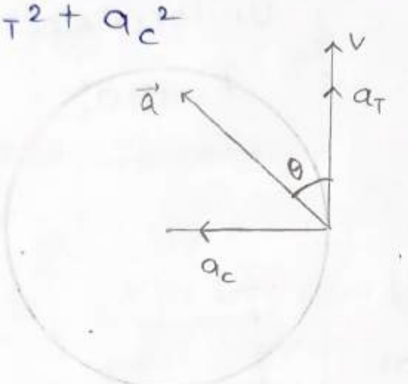


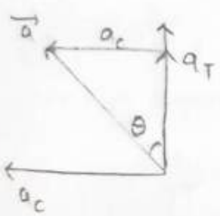
* Speed ↑ $(\theta < 90^\circ)$

$|\vec{a}| = \sqrt{a_T^2 + a_c^2}$

↓

always





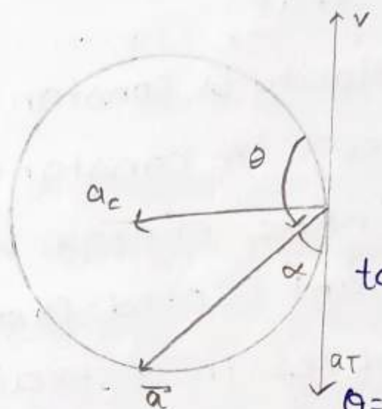
$\tan\theta = \frac{a_c}{a_T}$

Angle b/w $\vec{a}$ & $\vec{v}$

$\cdot\ |\vec{a}_c|_{inst} = \frac{v^2}{R}$

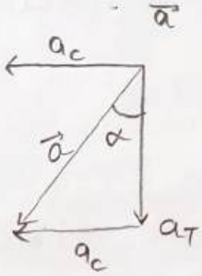
$\cdot\ \vec{a}_t = \frac{d|\vec{v}|}{dt}$

* Speed ↓ $(\theta > 90^\circ)$



$\tan\alpha = \frac{a_c}{a_T}$

$\theta = 180 - \alpha$

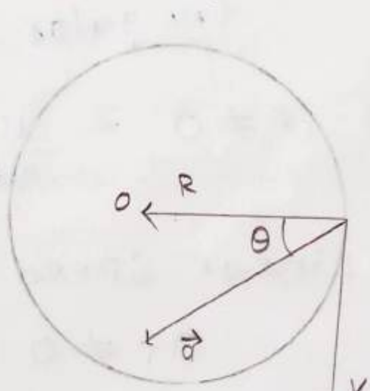


$|\vec{a}| = \sqrt{a_T^2 + a_c^2}$

Que - find a_c and a_T

→ $a_c = a\cos\theta = \frac{v^2}{R}$

→ $a_T = a\sin\theta = \frac{d|\vec{v}|}{dt} = R\alpha$



Que- A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'a' m/sec², then the resultant acceleration will be

(a) $\sqrt{\frac{v^2}{r^2} - a^2}$

✓(b) $\sqrt{\frac{v^4}{r^2} + a^2}$

(c) $\sqrt{\frac{v^4}{r^2} - a^2}$

(d) $\sqrt{\frac{v^2}{r^2} + a^2}$

MR* U.C.M

$a = a_c$

$a_T = 0$

N.U.C.M.

$\vec{a} = \sqrt{a_T^2 + a_c^2}$

$a_T = a$ —— (i)

$a_c = v^2/R$ —— (ii)

$|\vec{a}| = \sqrt{a_T^2 + a_c^2} \rightarrow \sqrt{a^2 + \frac{v^4}{R^2}}$

Que- A body is moving on a circle of radius 80 m with a speed 20 m/s which is decreasing at the rate 5 m/s² at an instant. The angle made by its acceleration with its velocity is

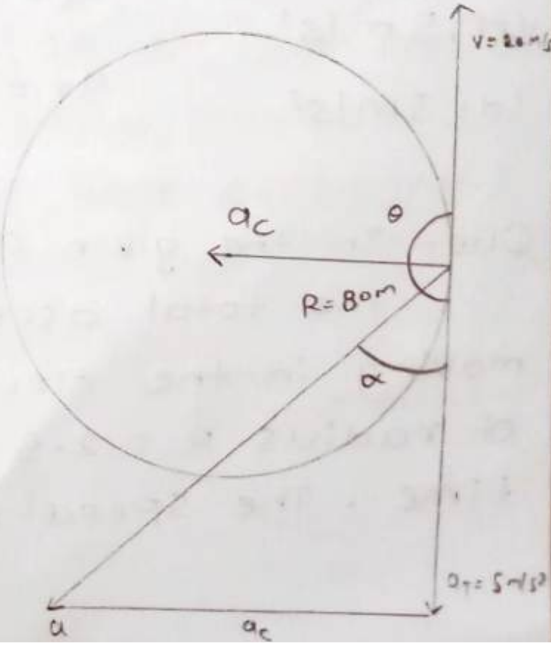
(a) 45°

(b) 90°

✓(c) 135°

(d) 0°

$\frac{400}{80} \rightarrow \frac{(20)^2}{80} = \frac{v^2}{R}$



Que- The distance of a particle moving on a circle of radius 12m measured from a fixed point on the circle and measured along the circle is given by $S = 2t^3$ (in meters). The ratio of its tangential to centripetal acceleration at t = 2 sec

(a) 1 : 1
(b) 1 : 2 ✓
(c) 2 : 1
(d) 3 : 1

Speed: $V = \frac{ds}{dt} = 2\frac{d(t^3)}{dt}$

$V = 6t^2$

$a_T = \frac{dv}{dt} = 6\frac{dt^2}{dt} = 12t$

$a_c = \frac{V^2}{R} = \frac{(6t^2)^2}{R} = \frac{36t^4}{12}$

$= 3t^4$

$\frac{a_T}{a_c} = \frac{12t}{3t^4} = \frac{4}{t^3} = \frac{4}{(2)^3}$

$= \frac{4}{8} = \frac{1}{2}$

Que - A car is moving at a speed of 40 m/s on a circular track of radius 400 m. This speed is increasing at the rate of $3 m/s^2$. The acceleration of car is

(a) $4 m/s^2$
(b) $7 m/s^2$
(c) $5 m/s^2$ ✓
(d) $3 m/s^2$

$V = 40 m/s$
$R = 400 m$
$a_c = V^2/R = \frac{40 \times 40}{400}$
$a_T = 3 m/s^2$

N.U.C.M.

$a = \sqrt{a_T^2 + a_c^2}$
$= \sqrt{(4)^2 + (3)^2}$
$= \sqrt{25}$
$= 5$

Que- In the given figure, $a = 15 ms^{-2}$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is

(a) $4.5\ ms^{-1}$

(b) $5.0\ ms^{-1}$

✓(c) $5.7\ ms^{-1}$

(d) $6.2\ ms^{-1}$

$$\frac{15\sqrt{3}}{2} = \frac{V^2}{(5/2)}$$

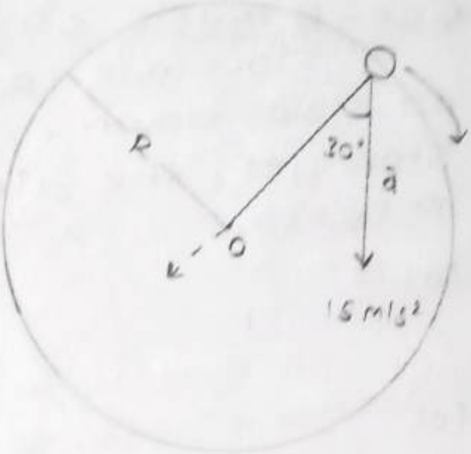
$$V^2 = \frac{5}{2} \times \frac{15\sqrt{3}}{2}$$

$$= \frac{75}{4}\sqrt{3}$$

$$= 19\sqrt{3}$$

$$V^2 = 19 \times 1.71 = 32$$

$$V = \sqrt{32} = 5.7$$



$$a_c = (a\cos 30°) = 15 \times \frac{\sqrt{3}}{2}$$

$$= \frac{V^2}{R}$$

Que- If θ is angle between the velocity and acceleration of a particle moving on a circular path with decreasing speed, then

(a) $\theta = 90°$

(b) $0° < \theta < 90°$

✓(c) $90° < \theta < 180°$

(d) $0° \le \theta \le 180°$

(e) $90° \le \theta \le 180°$

Que- A motor car is travelling at 30 m/sec on a circular road of radius 500 m. It is increasing its speed at the rate of $2.0\ ms^{-2}$. The total acceleration is

(a) $1.8\ ms^{-2}$

(b) $2\ ms^{-2}$

(c) $3.8\ ms^{-2}$

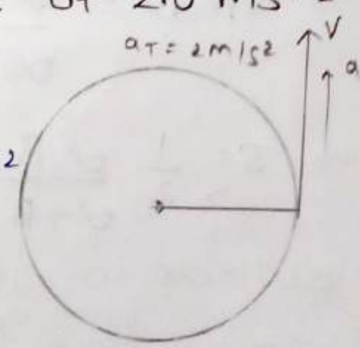
✓(d) $2.7\ ms^{-2}$

$$a_c = \frac{V^2}{R} = \frac{(30)^2}{500} = \frac{900}{500} = \frac{9}{5} \simeq 2$$

$$|\vec{a}| = \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{8} = 2\sqrt{2}$$

$$= 2 \times 1.4$$



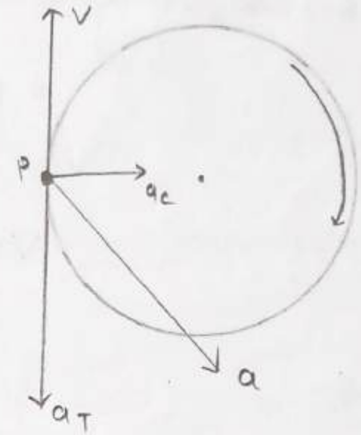
Que- A music CD of 'Bajirao Mastani' is rotating clockwise (as shown). After turning it off, the CD slows down. Assuming it has not come to a stop yet, the direction of acceleration at point P is

(a) 

(b) 

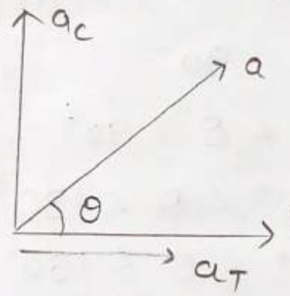
(c) 

✓(d) 



* Equation of Motion

$$\vec{V_f} = \vec{u_i} + \vec{a_t} t \longrightarrow \text{1-D Motion}$$



$$\vec{S} = \vec{u_t} t + \frac{1}{2} a_t t^2$$

$$V^2 - u^2 = 2a_t S$$

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

Stopping distn. $S = \dfrac{u^2}{2a}$

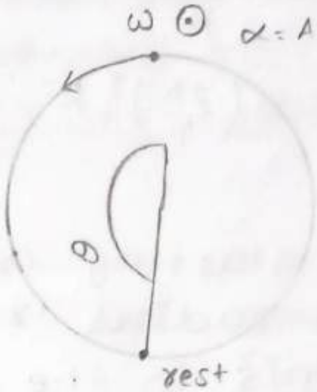
→ $V_{max} = \dfrac{\alpha\beta}{\alpha+\beta} T$ (Rest)

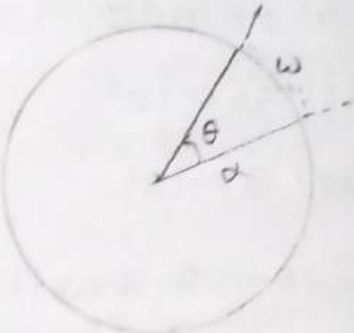
→ $S = \dfrac{1}{2} \dfrac{\alpha\beta}{\alpha+\beta} T^2$

* Equation of motion in circular Motion

$$\alpha = \frac{d\omega}{dt}$$

$$\int_0^t \alpha\, dt = \int_{\omega_i}^{\omega_f} d\omega$$





If $\alpha = \text{cost}^n$

$$\alpha t = \omega_f - \omega_i$$

$$\omega_f = \omega_i + \alpha t \quad \text{— (i)}$$

$$\theta = \vec{\omega_i} t + \frac{1}{2}\alpha t^2 \quad \text{— (ii)}$$

$$\omega_f^2 - \omega_i^2 = 2\alpha\theta \quad \text{— (iii)}$$

$$\theta_{th} = \omega_i + \frac{\alpha}{2}(2n-1) \quad \text{— (iv)}$$

Stopping Angle

$$\theta = \frac{\omega_i^2}{2\alpha}$$

U.C.M.

$\alpha = 0$, $a_T = 0$

$\vec{\omega_f} = \vec{\omega_i}$

$\theta = \omega_i t$

$$a_c = \frac{V^2}{R}$$

N.U.C.M.

→ $\alpha = 2\ \text{rad/sec}^2$

→ $\omega_i = 4\ \text{rad/sec}$ then find angular speed after $t = 3$ sec

$\vec{\omega_f} = \vec{\omega_i} + \alpha t$

$\omega_f = 4 + 2 \times 3$

$= 10\ \text{rad/sec}$

Que- A particle moves in a circle of radius 5 cm with constant speed and time period $0.2\pi s$. The acc^n of the particle is

(a) $15\ m/s^2$ $\quad T = 0.2\pi = 2\pi/10$, $R = 5\ cm$

(b) $25\ m/s^2$ $\quad a_c = \frac{V^2}{R} = \omega^2 R$

(c) $36\ m/s^2$ $\quad a_c = \left(\frac{2\pi}{T}\right)^2 R \rightarrow \left(\frac{2\pi}{\frac{2\pi}{T}}\right)^2 \times \frac{5}{100} = 5$

✓(d) $5\ m/s^2$

Que- A particle starting from rest moves in a circle of radius 'r'. It attains a velocity of V_0 m/s in the n^{th} round. Its angular acceleration will be

(a) $\frac{V_0}{n}$ rad/s²

(b) $\frac{V_0}{2\pi n r^2}$ rad/s²

✓(c) $\frac{V_0^2}{4\pi n r^2}$ rad/s²

(d) $\frac{V_0^2}{4\pi n r}$ rad/s²

N.U.C.M.

$u_i = 0$

$\omega_i = 0$

$u_f = u_0$

$\omega_f = \frac{V_0}{r}$

$\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$\left(\frac{V_0}{r}\right)^2 - 0 = 2\alpha\theta$

$\frac{V_0^2}{r^2} = 2\alpha(2n\pi)$

$\alpha = \frac{V_0^2}{4n\pi r^2}$

Que- The position vector of a particle $\vec{R}$ as a function of time is given by $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$, where R is in meters, t is seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x- and y-directions, respectively. Which one of the following statements is wrong for the motion of particle?

(a) Path of the particle is a circle of radius 4m.

(b) Acceleration vector of along $-\vec{R}$

(c) Magnitude of acceleration vector is V^2/R, where v is the velocity of particle.

(d) Magnitude of the velocity of particle is 8 m/s

Que. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$, where ω is a constant. Which of the following is true

(a) Velocity is perpendicular to $\vec{r}$ and acceleration is directed away from the origin.

(b) velocity and acceleration both the perpendicular to $\vec{r}$

(c) velocity and acceleration both are parallel to $\vec{r}$

(d) velocity is perpendicular to $\vec{r}$ and acc^n is directed towards the origin.

Que - If the eq^n for angular disp^m of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2s from its start is

(a) 8 rad/s

(b) 12 rad/s

(c) 24 rad/s

(d) 36 rad/s

$$\theta = 2t^3 + 0.5$$

$$\omega = \frac{d\theta}{dt} = \frac{2dt^3}{dt}$$

$$\omega = 2 \times 3t^2$$

$$\omega = 6t^2$$

$$\omega = 6 \times (2)^2$$

$$= 24$$

Que - A particle of mass 'm' describes a circle of radius 'r'. The centripetal accn of the particle is $4/r^2$. The momentum of particle

(a) $2m/r$
✓(b) $2m/\sqrt{r}$
(c) $4m/r$
(d) $4m/\sqrt{r}$

$a_c = \frac{4}{r^2} = \frac{v^2}{r}$

$v^2 = \frac{4}{r}$

$v = \sqrt{\frac{4}{r}} = \frac{2}{\sqrt{r}}$

$P = mv$

$P = m \times \frac{2}{\sqrt{r}}$

$P = \frac{2m}{\sqrt{r}}$

Que - A stone tied to the end of a string of 1m long is whirled in a horizontal circle with a constant speed. If the stone make 22 revolution in 44 seconds, what is the magnitude and direction of accn of the stone?

✓(a) $\pi^2\ ms^{-2}$ and direction along the radius towards centre
(b) $\pi^2\ ms^{-2}$ and direction along the radius away from centre
(c) $\pi^2\ ms^{-2}$ and direction along the tangent to the circle.
(d) $\pi^2/4\ ms^{-2}$ and direction along the radius towards the centre.

$a_c = \omega^2 R = (2\pi f)^2 R \Rightarrow \left(2\pi \frac{1}{2}\right)^2 \times 1 \Rightarrow \pi^2$

$f = \frac{22\ r}{44\ s} = \frac{1}{2}\ r/s$

Que - A particle moves along a circle of radius $(20/\pi)$ m with constant tangential accn. If the velocity of the particle is 80 m/s at the end of second revolution after motion has begun, the tangential accn is

(a) $40 \, m/s^2$

(b) $640\pi \, m/s^2$

(c) $160\pi \, m/s^2$

(d) $40\pi \, m/s^2$

N.U.C.M.

$r = \frac{20}{\pi}$

2 rev. $(\theta = 4\pi)$

$v = 80 \, m/s$

$\omega_i = 0$, $\omega_f = \frac{v}{r}$

$= \frac{80\pi}{20}$

$= 4\pi$ rad/s

$\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$\cancel{16}\pi^2 = \cancel{2}\alpha \cancel{4}\pi$

$\alpha = 2\pi$

$a_T = r\alpha$

$= \frac{20}{\pi} \times 2\pi$

$= 40 \, m/s^2$

MR*

$v_f^2 - v_i^2 = 2 a_T S$

$(80)^2 = 2a_T \, 4\pi\left(\frac{20}{\pi}\right)$

$80 \times 80 = 8 \times 20 \times a_T$

$a_T = 40 \, m/s^2$

Que- A car moves on a circular path such that its speed is given by $v = kt$, where k = constant and t is time. Also given: radius of the circular path is r. The net acceleration of the car at time 't' will be

(a) $\sqrt{k^2 + \left(\frac{k^2t^2}{r}\right)^2}$

(b) $2k$

(c) k

(d) $\sqrt{k^2 + k^2t^2}$

$v = kt$

N.U.C.M.

$a_c = \frac{v^2}{r}$

$a_t = \frac{dv}{dt}$

$a_c = \frac{k^2t^2}{r}$

$a_t = k$

$a = \sqrt{\frac{k^4t^2}{r^2} + k^2}$

Que- The radius vector of a particle moving on a circle is given by $\vec{r} = A\cos Bt \, \hat{i} + A\sin Bt \, \hat{j}$ (A and B are constants). The radius of the circle and speed of the particle, respectively are

(a) A, AB

(b) A, A^2/B

(c) B, AB

(d) B, A^2/B

$$\vec{r} = A\cos(Bt)\,\hat{i} + A\sin(Bt)\,\hat{j}$$

$$\vec{r} = R\cos(\omega t)\,\hat{i} + R\sin(\omega t)\,\hat{j}$$

$\omega t = Bt \rightarrow$ $v = r\omega$

$\omega = B$ $\quad$ $v = AB$

Que- A particle starts moving on a circular path from rest, such that its tangential acceleration varies with time as $a_t = kt$. Distance travelled by particle on the circular path in time 't' is

(a) $\frac{kt^3}{3}$

(b) $\frac{kt^2}{6}$

(c) $\frac{kt^3}{6}$

(d) $\frac{kt^2}{2}$

$v_i = 0$

$\omega_i = 0$

$a_t = kt \rightarrow$ variable tangential acceleration

$$\frac{dv}{dt} = kt$$

$$\int_0^v dv = \int_0^t kt\,dt$$

$$v = \frac{kt^2}{2}$$

Speed

$$\frac{ds}{dt} = \frac{k}{2}t^2$$

$$\int ds = \frac{k}{2}\int t^2\,dt$$

$$\text{Dist}^n = \frac{kt^3}{6}$$

Que- A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration, if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion?

(a) $0.15\ m/s^2$

$m = 10\,g$

$R = 6.4\,cm$

$a_T =$ constant (N.U.C.M.)

(b) 0.18 m/s²	$a = \sqrt{a_T^2 + a_c^2}$	$K.E. = 8 \times 10^{-4} = \frac{1}{2} mv^2$
(c) 0.2 m/s²	$\left.\begin{array}{l}\theta = 4\pi \\ Dist^n = 4\pi R\end{array}\right]$	
(d) 0.1 m/s²		